CIEAEM65 *in honour of Emma Castelnuovo*
Torino, Italy 22 - 26 July 2013

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Univ. Di Roma "La Sapienza"

In a short time, and with
a first rough approximation,
I can characterize the teaching
of Emma Castelnuovo [EC]
in the transition from
a **smaller** to a **greater** presence of:

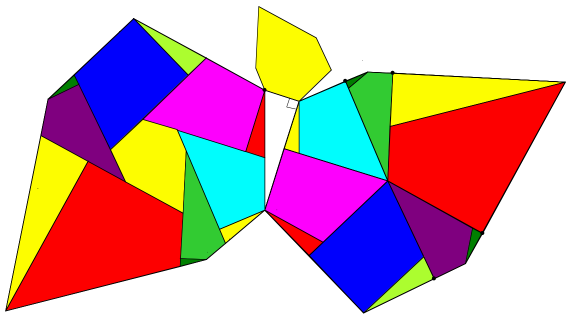
deductive teaching	**inductive teaching**
axiomatic approach	**"natural" approach**
abstract background	**concrete background**
static teaching	**dynamic teaching**
descriptive teaching	**constructive teaching**
routine	**reasoning**
repetition	**participation, discovery**
"more" Arithmetic and Algebra	**"more" Geometry**
many calculations	**few calculations**
words	**drawings, materials [no one like EC]**
"ugliness" and lack of color	**beauty and colours**
boring subjects	**interesting topics**
unclear aims	**important applications**
topics little related	**connections between the topics**
infinitesimal calculus	**infinitesimal reasoning**
perfection that is illusory	***approximation, that is reality***
coldness	**affection, seduction, empathy**

Shortly:

- **The "tradition" starts from mathematics, trying to teach it well to the student**

- **Emma starts from the student and look for the mathematics that can be useful to him, through an operative, efficient, and convincing teaching, looking at his needs and those of society**

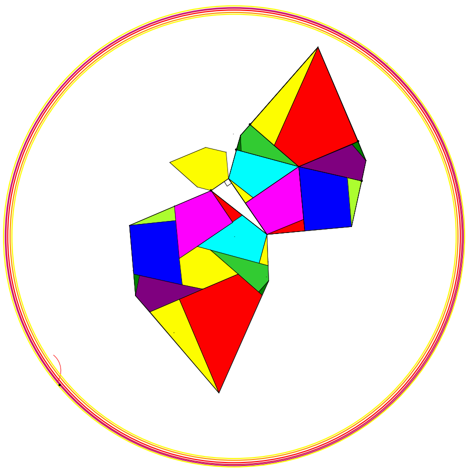
how I see EMMA

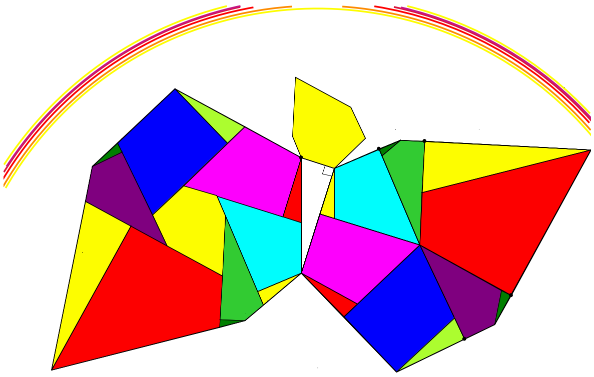


color, movement, beauty, lightness, "adaptability", game, ..., and mathematics

1) There are the colors of peace

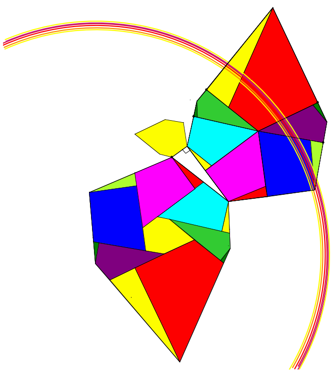
2) the butterfly is lightweight



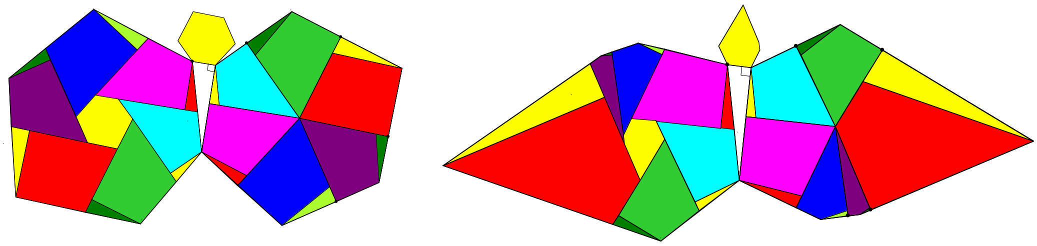


3) the butterfly can rotate

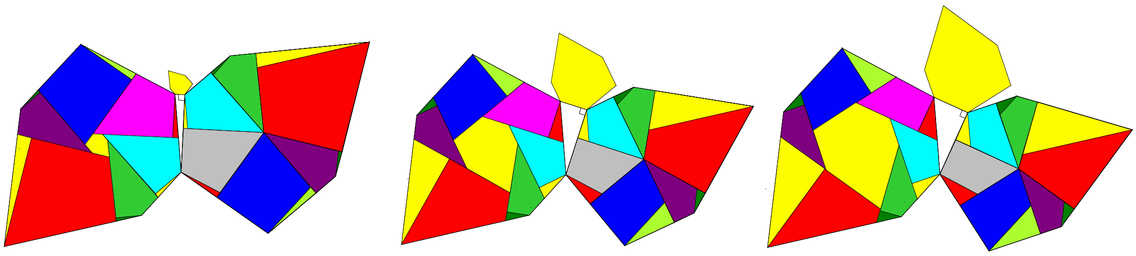
4) the butterfly can become bigger



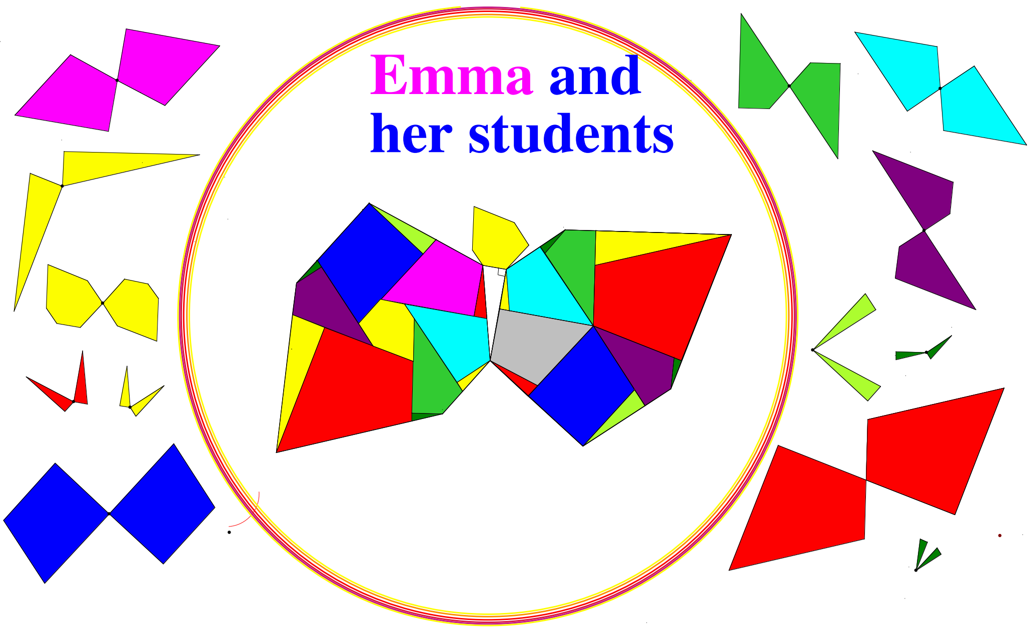
5) the butterfly can fly



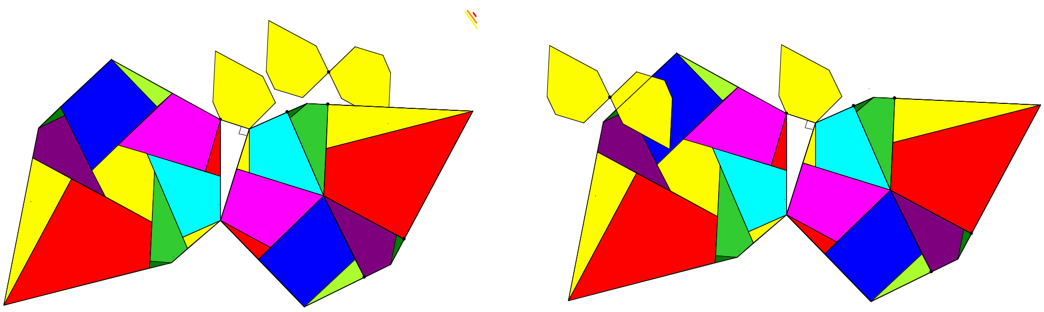
5) … can be modified with continuity
in any similar polygons that circumscribe a circle

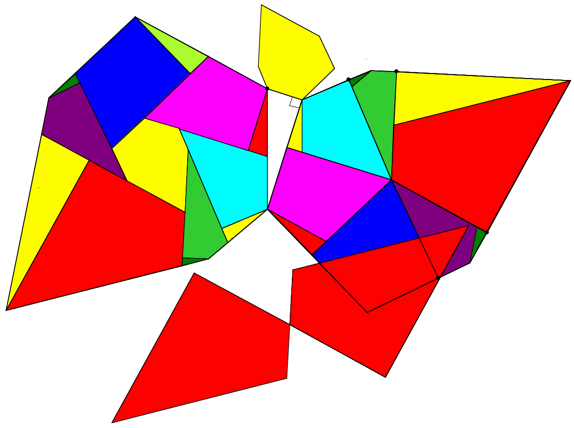


6) it shows a quite general
theorem of Pythagoras

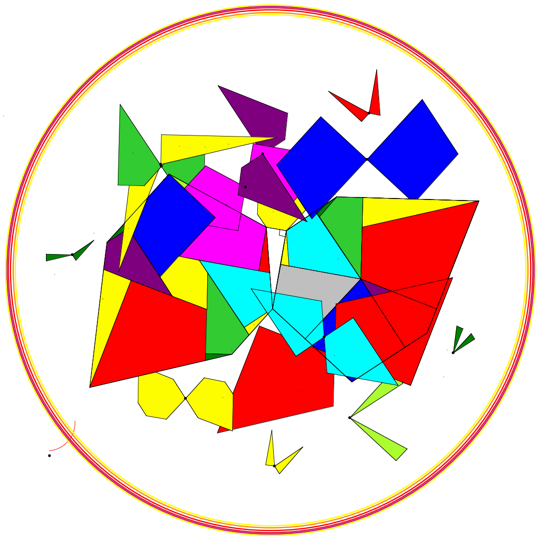


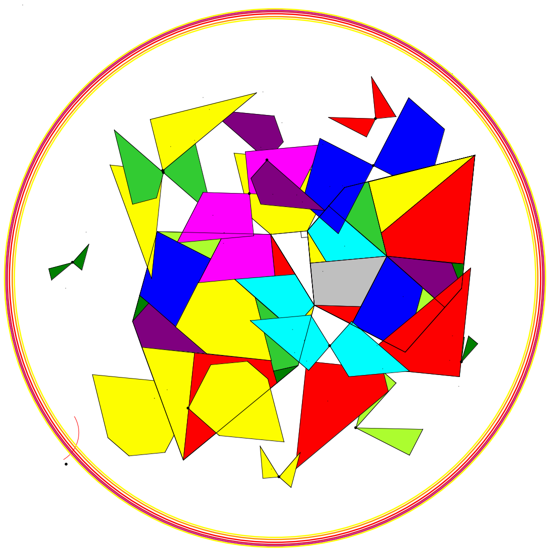
7) the butterfly is a puzzle





8) Emma is celebrated by her students

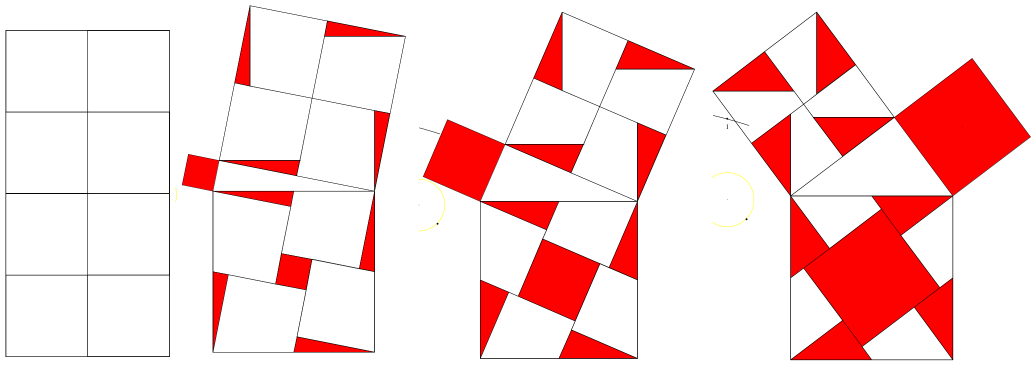




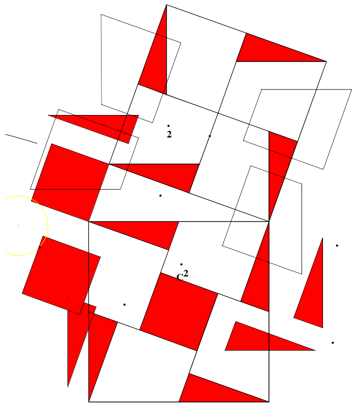
Emma's students know that:

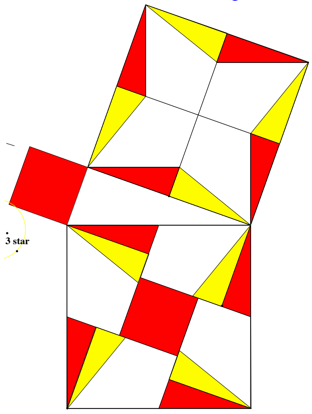
the rules of international law
and the values of peace
are the basis of freedom.
Freedom is founded on the meeting, the collaboration
and respect for history and different cultures.
In the culture of the world,
by the point of view of history, language,
content (absolutely common),
there is nothing more international
and more democratic than
mathematics.
Mathematics can develop students' skills
and respond to the needs of society
in the best way.

Emma's Students begin with the simplest cases and appreciate aesthetically the symmetry

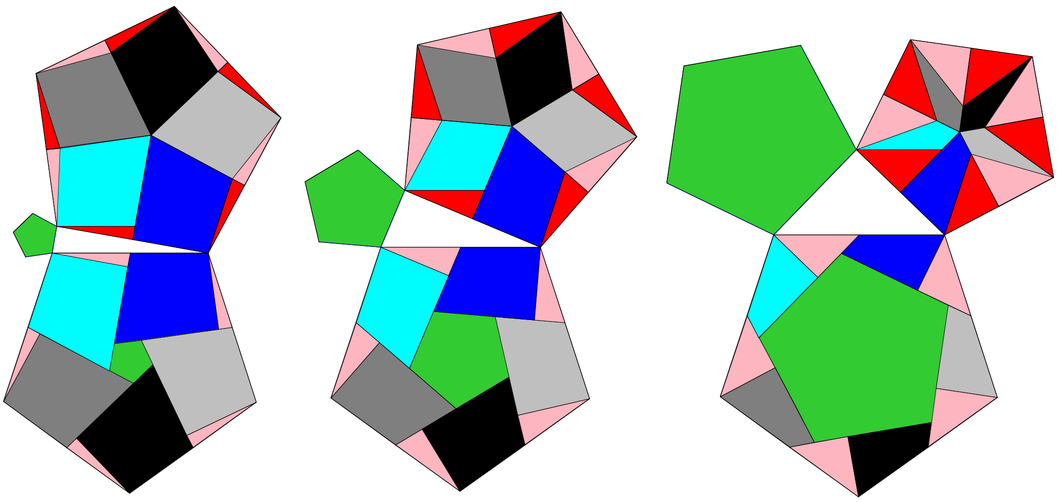


a simple puzzle: the pieces can be moved continuosly.

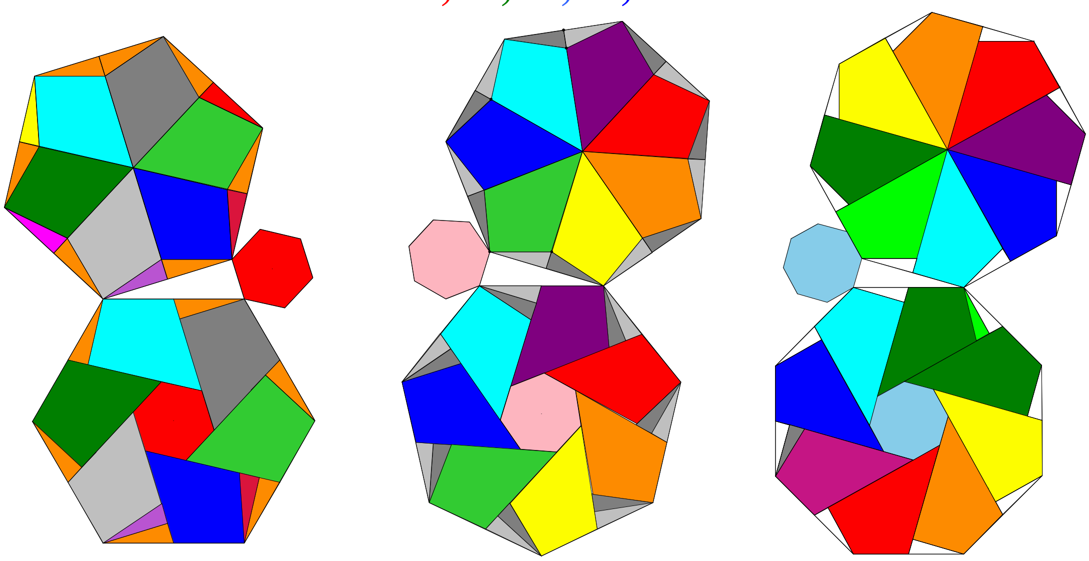




With the symmetry:

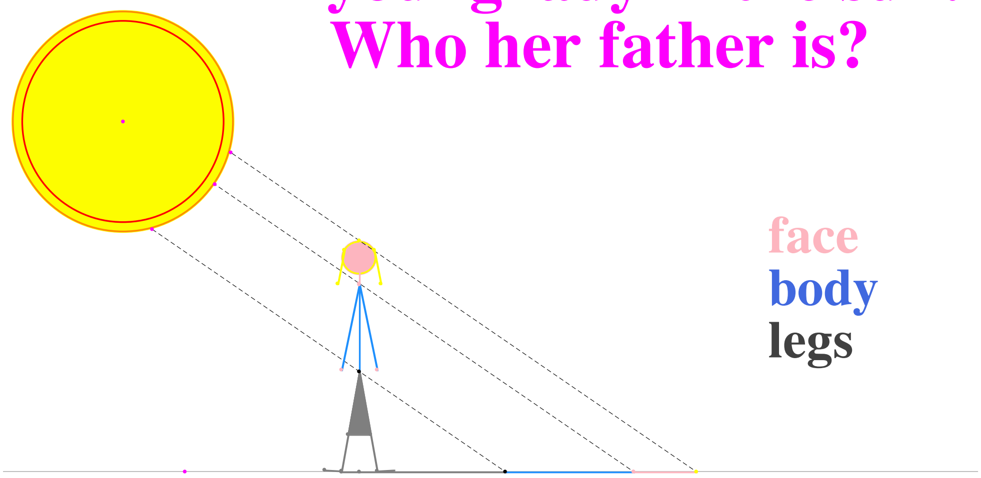


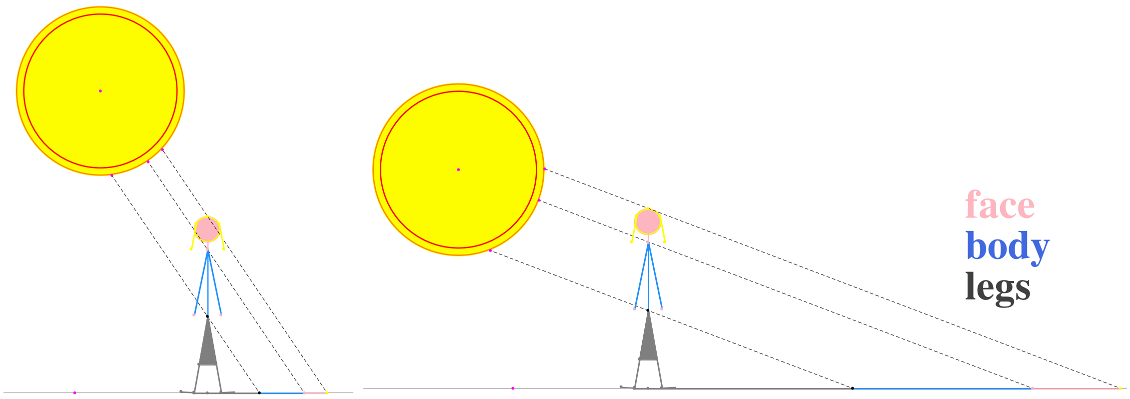
4, 5, 6, 7, 8



“To start from the student also in the communication” (Emma)

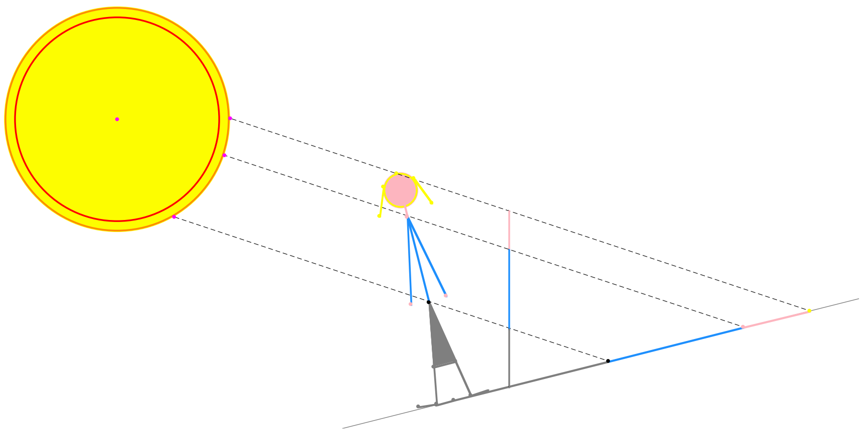
A young lady in the sun. Who her father is?





her father is Thales

- If the head is half of the body,
even the pink segment is half of the blue segment ...
- If the head doubles in the shade,
the body also doubles in the shade

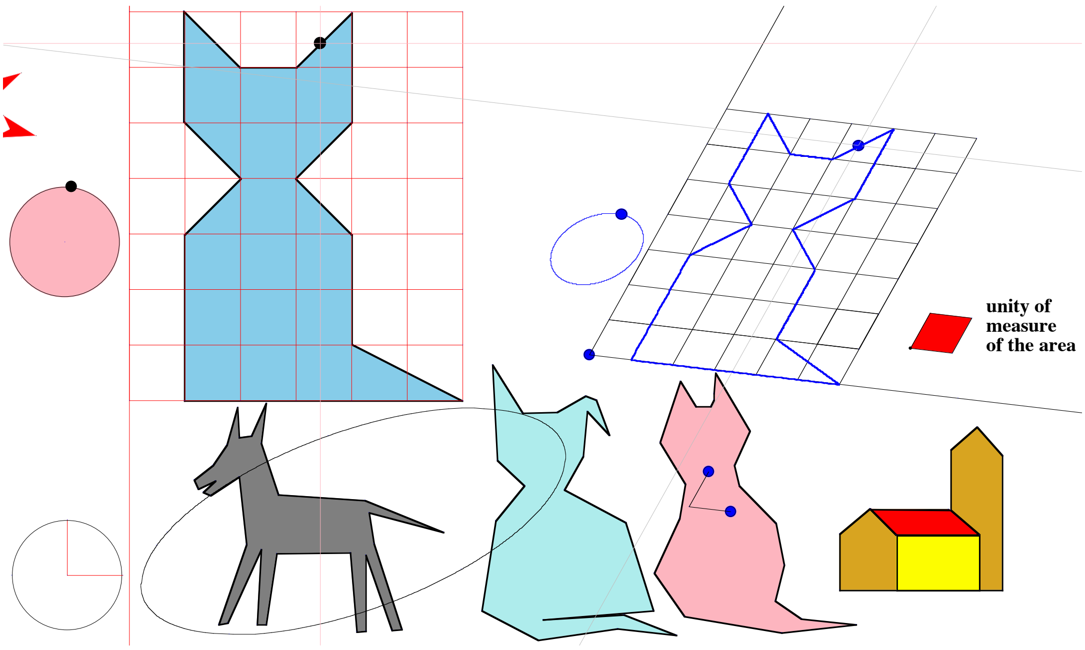


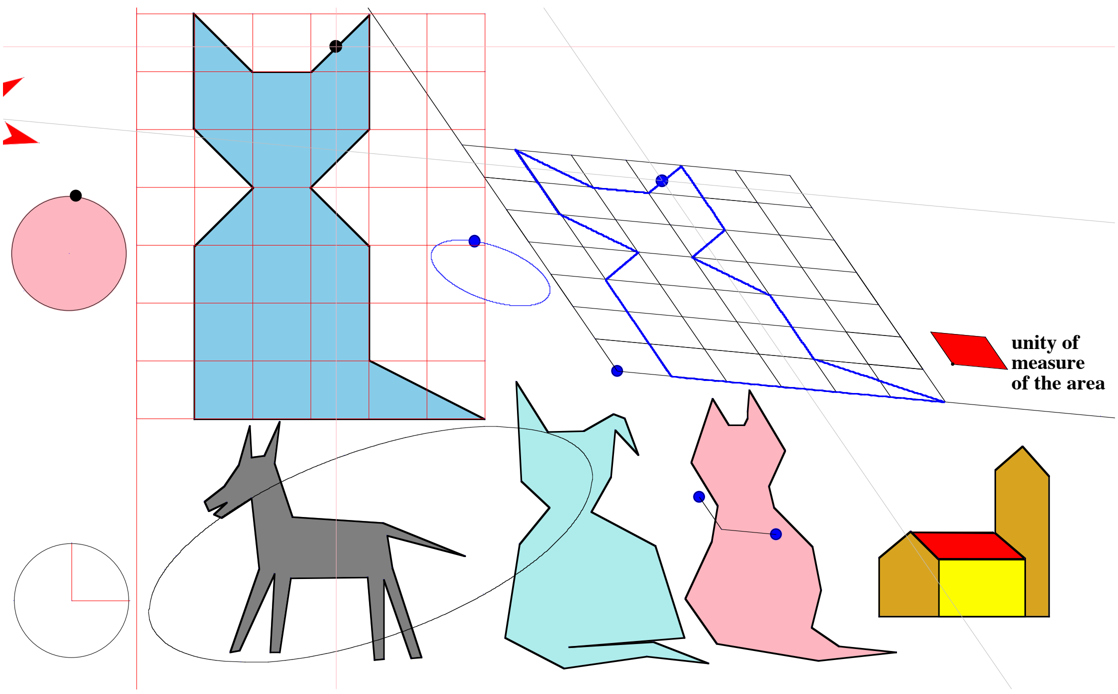
even if the ground is uphill …

“However” it needs to do something because
in the university the students of the V year of mathematics
don't know how to divide a segment
in 3 equal parts using a DGS (Cabri, GeoGebra, …)!

The girl's shade
shows an affinity on the straight line (TA1)

Now an affinity on the plan (TA2):





students can discover the invariants of
an affine transformation

invariants (trasf. aff.) TA2:

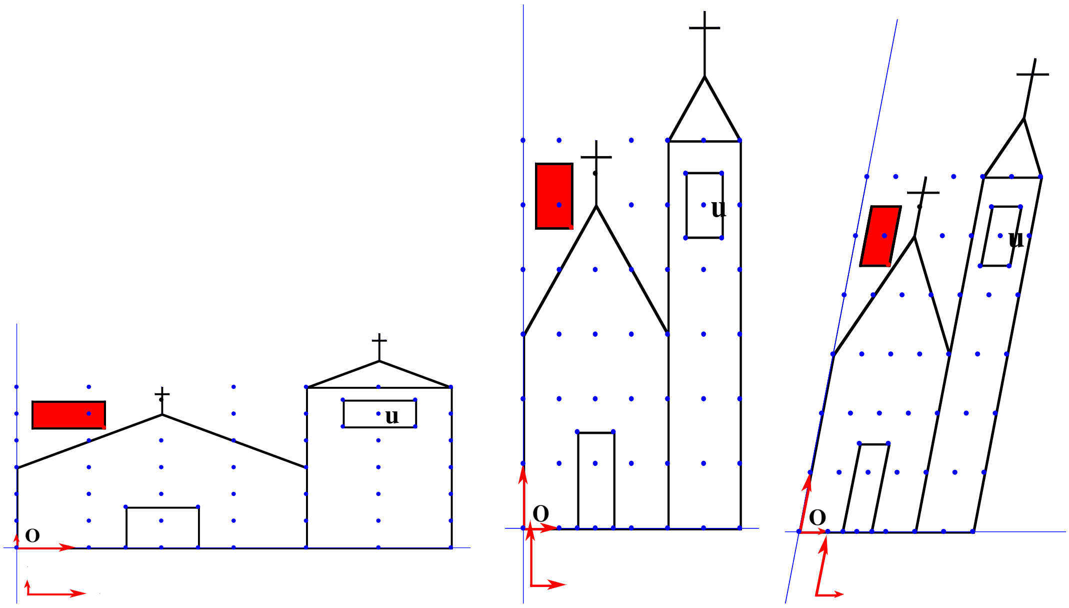
- Parallelism of lines

- Ratio of the areas

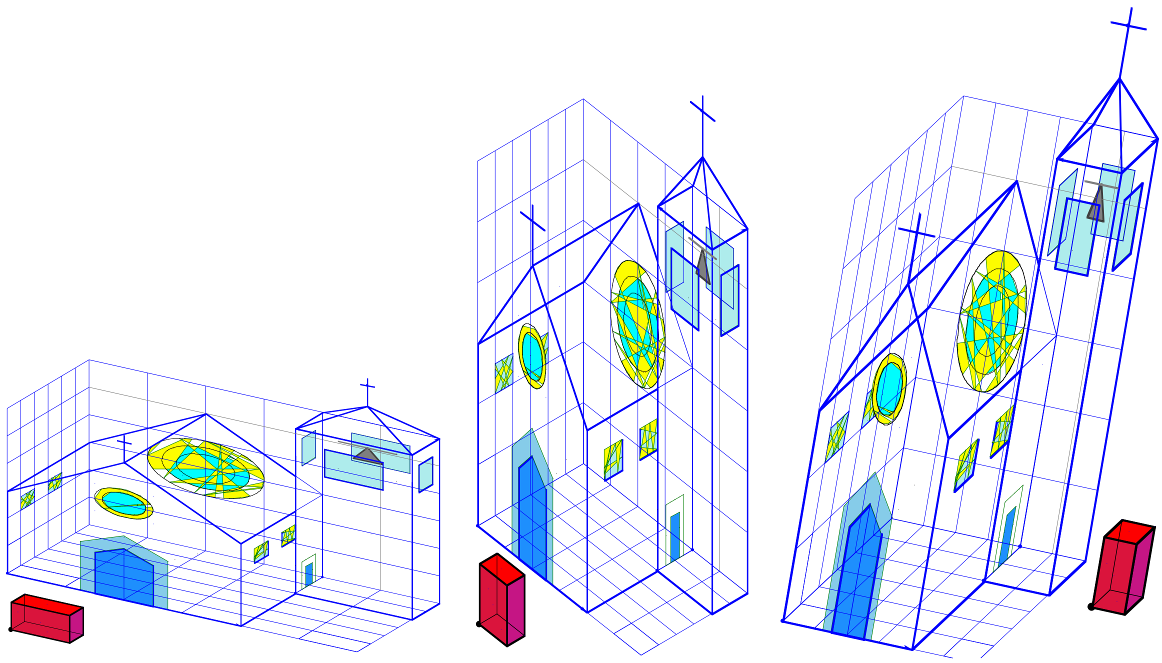
- A fixed ratio altering areas and the unit of measure

- Ratio of the lengths on a same straight line (TA1), or on parallel lines

- ...



The Romanic Church, the Gothic Church, the Church of Pisa



invariants (trasf. aff.) TA3:

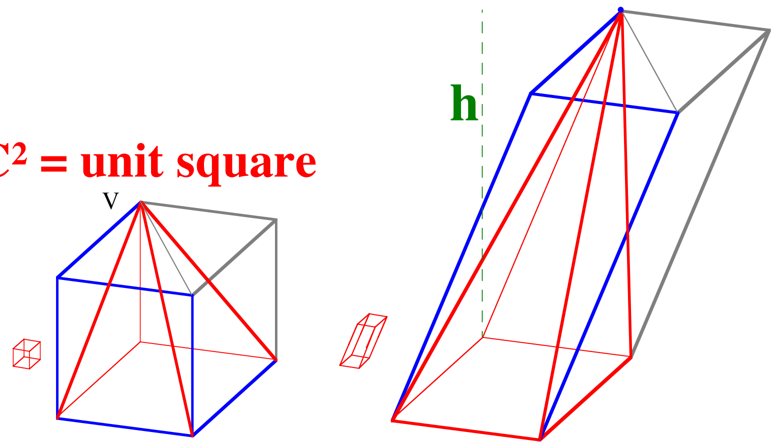
- parallelism - ratio of the areas on a same plan (TA2), or on parallel plans - ratio of the volumes -

...

An important application, forgetting Euclid

For a more systematic development of areas that immediately carries over to volumes in three or more dimensions, it is desirable to give a direct definition that is not tied to the idea of integration of functions of one variable and corresponds more closely to the intuitive notion of the area of a region as the 'number of square units' contained in the region.
Courant R., Fritz J., *Introduction to Calculus and Analysis, Vol. II*, Springer, p. 368 - 374.

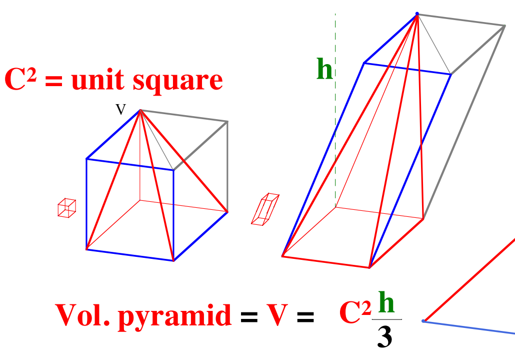
Volume of a solid with a "tip" (pyramid, cone, ...)

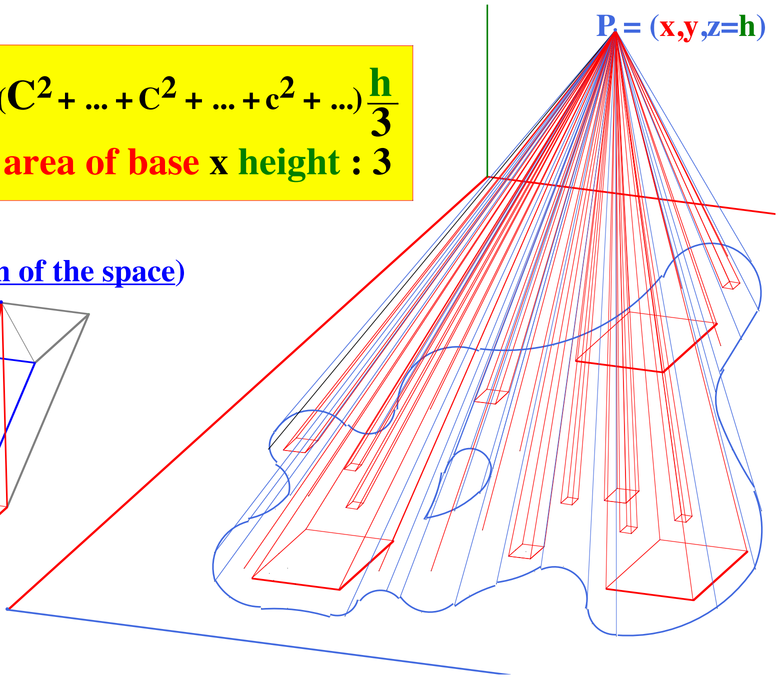


$$\text{Vol. pyramid} = V = C^2\frac{h}{3}$$

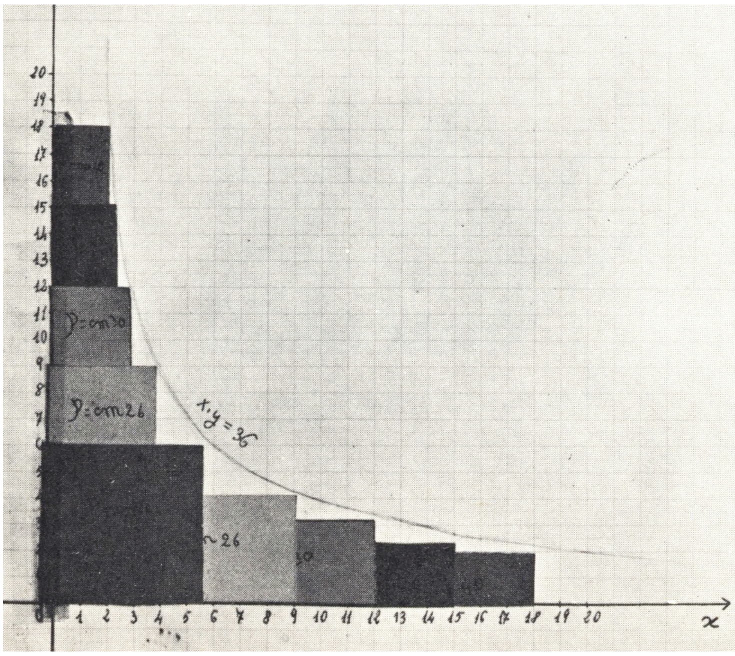
proofs:
- simple
- easily expressible
- viewable
- easily memorized
- reusable (eg. affinity)
- easy to generalize
(in this case in any dimension of the space)

$$V = (C^2 + \ldots + C^2 + \ldots + c^2 + \ldots)\frac{h}{3}$$
$$= \text{area of base} \times \text{height} : 3$$

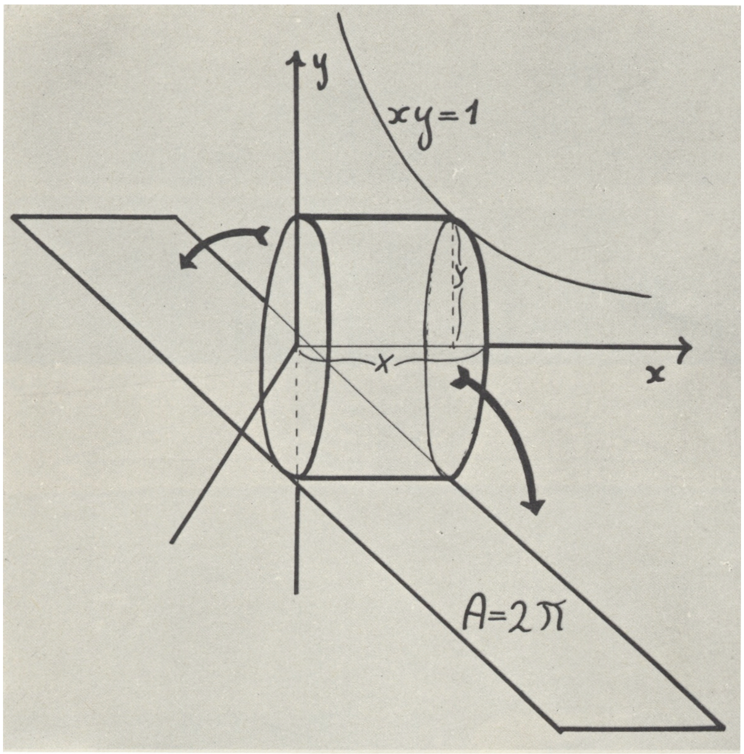




Learning Materials

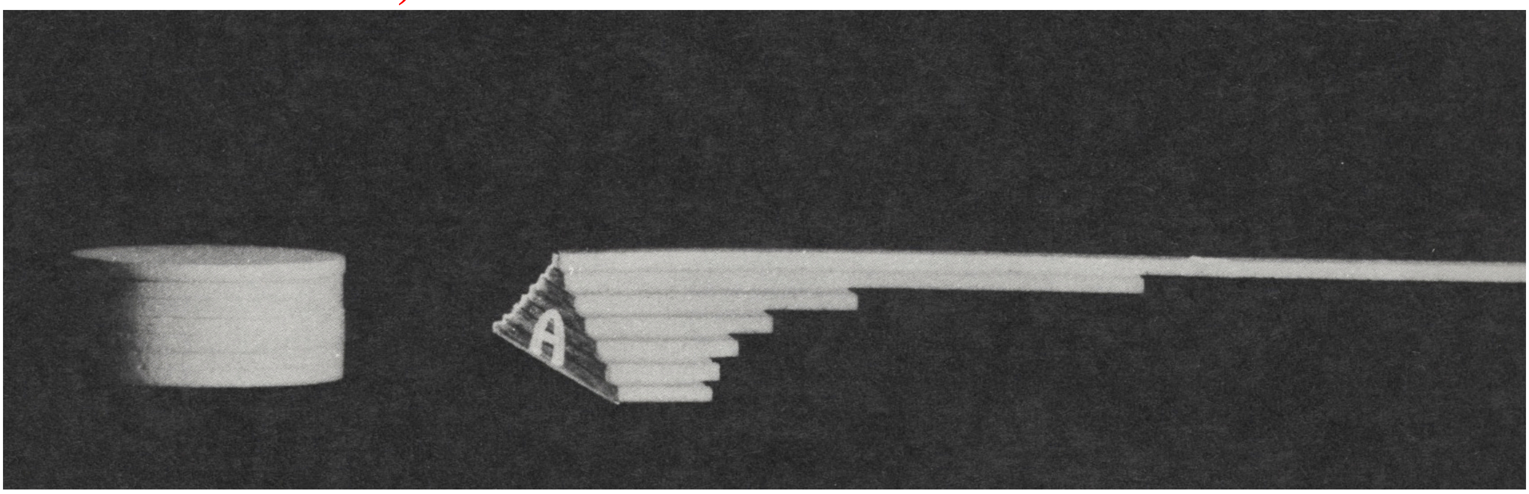


M1)

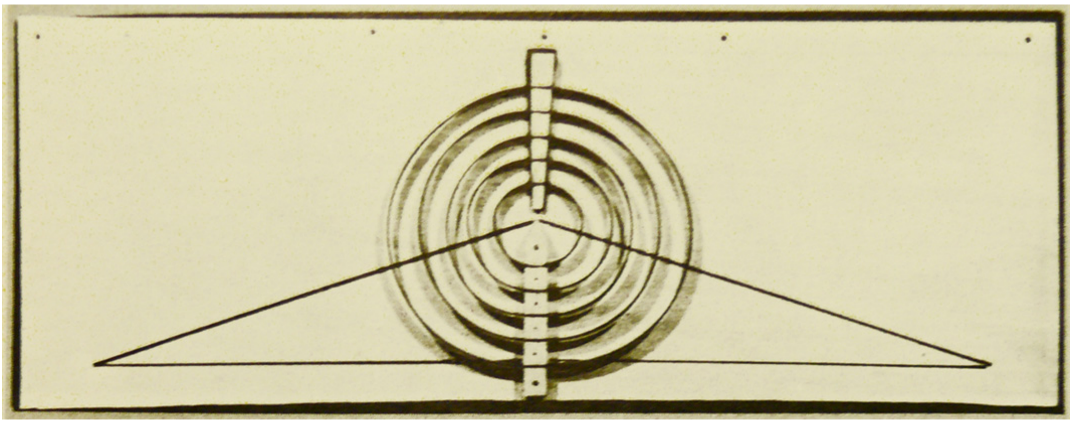


M1')

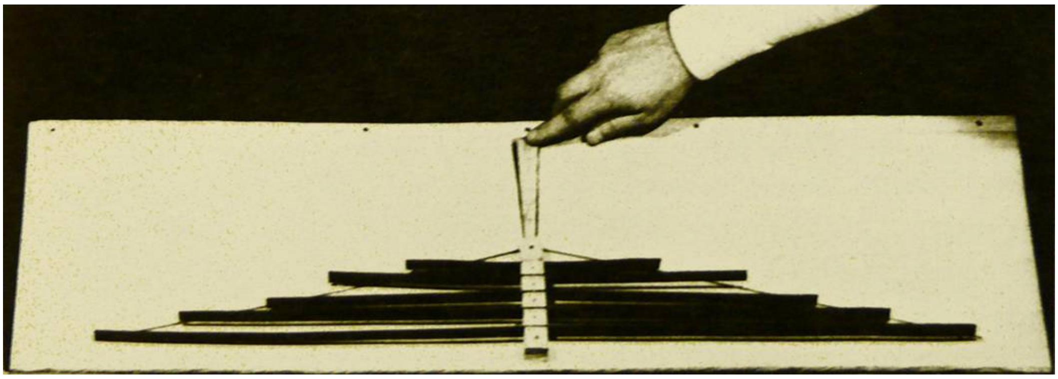
M1'')

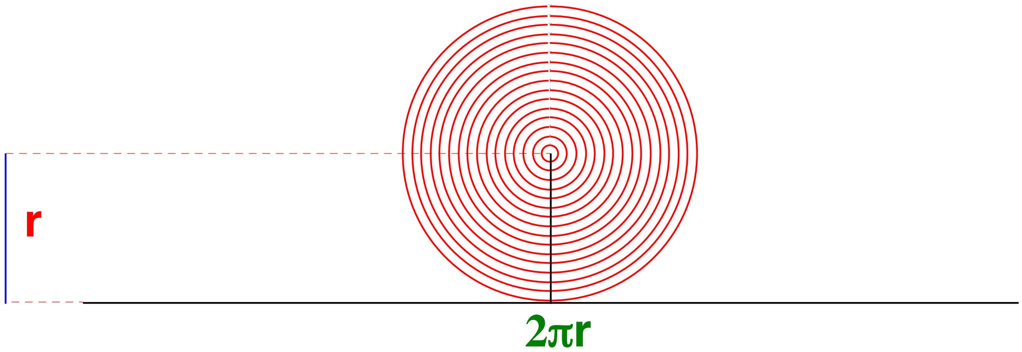


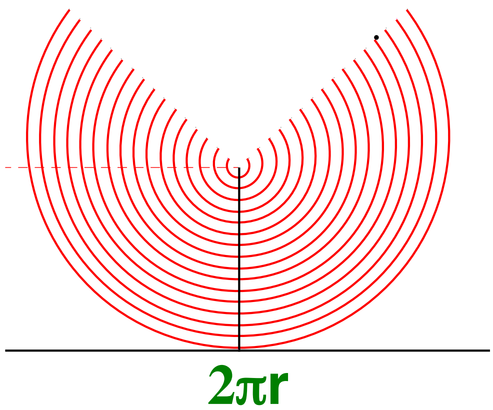
M(2)

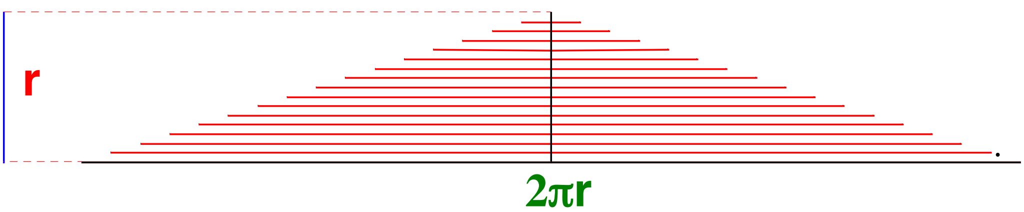


M2')

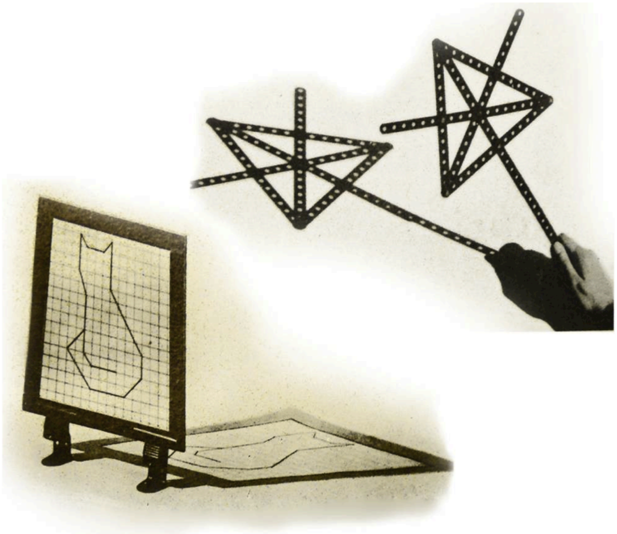






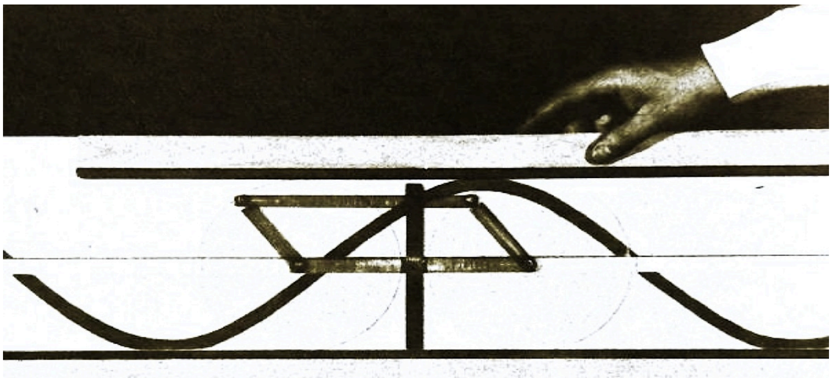


The fixed points are on the black segment

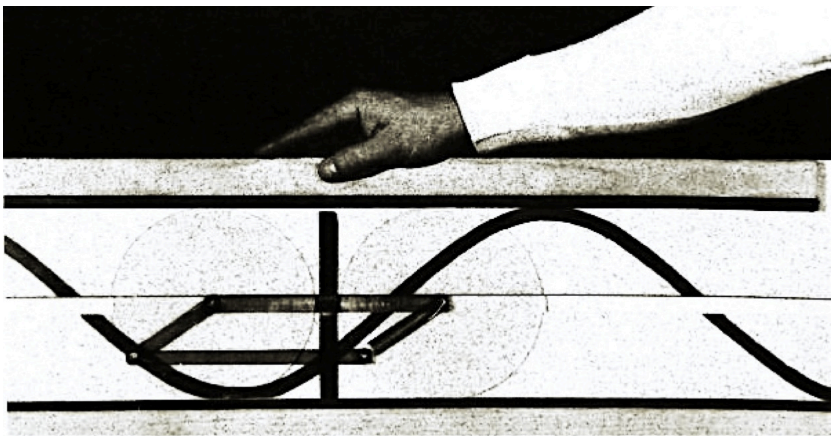


M3)

M4)

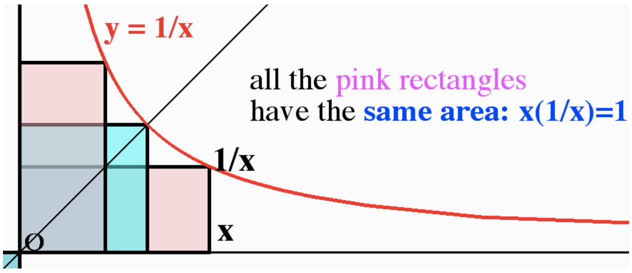


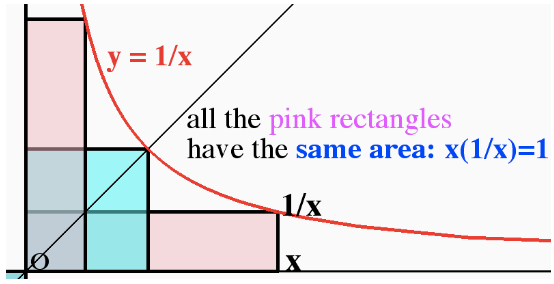
M4')



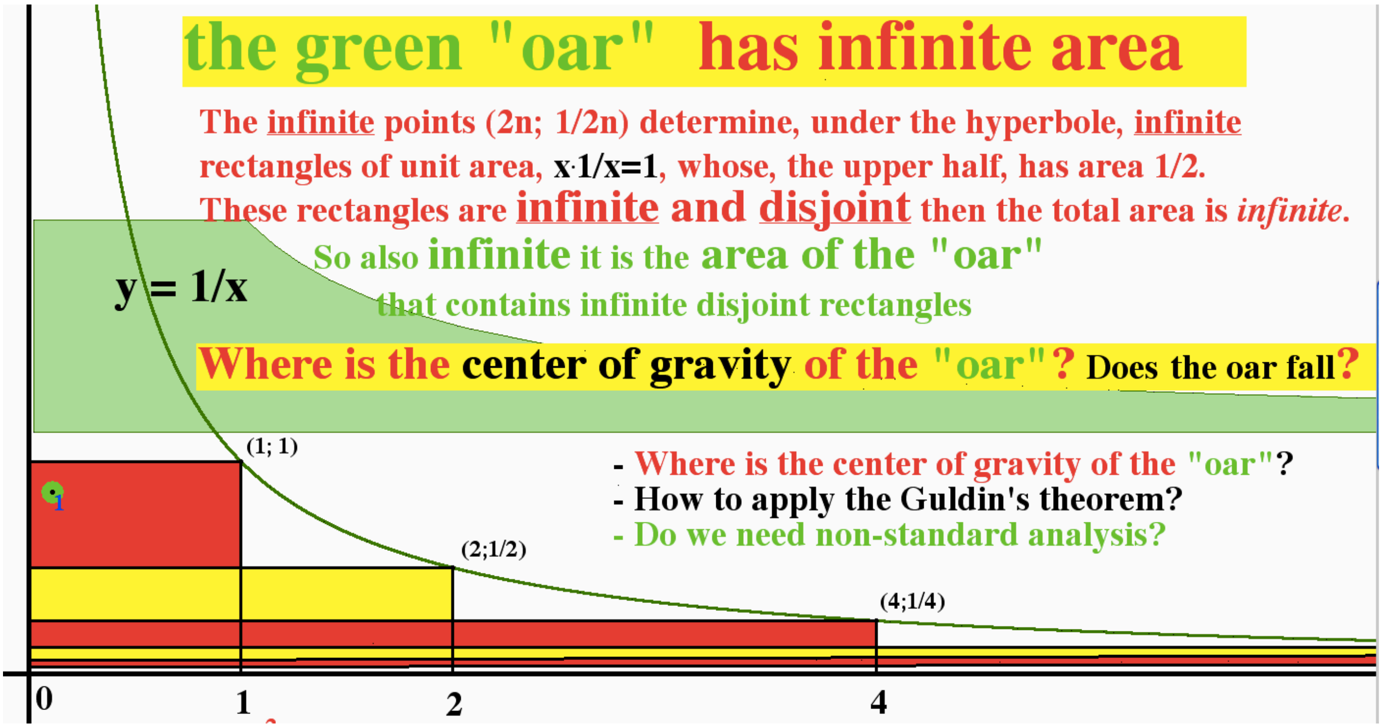
M4'')

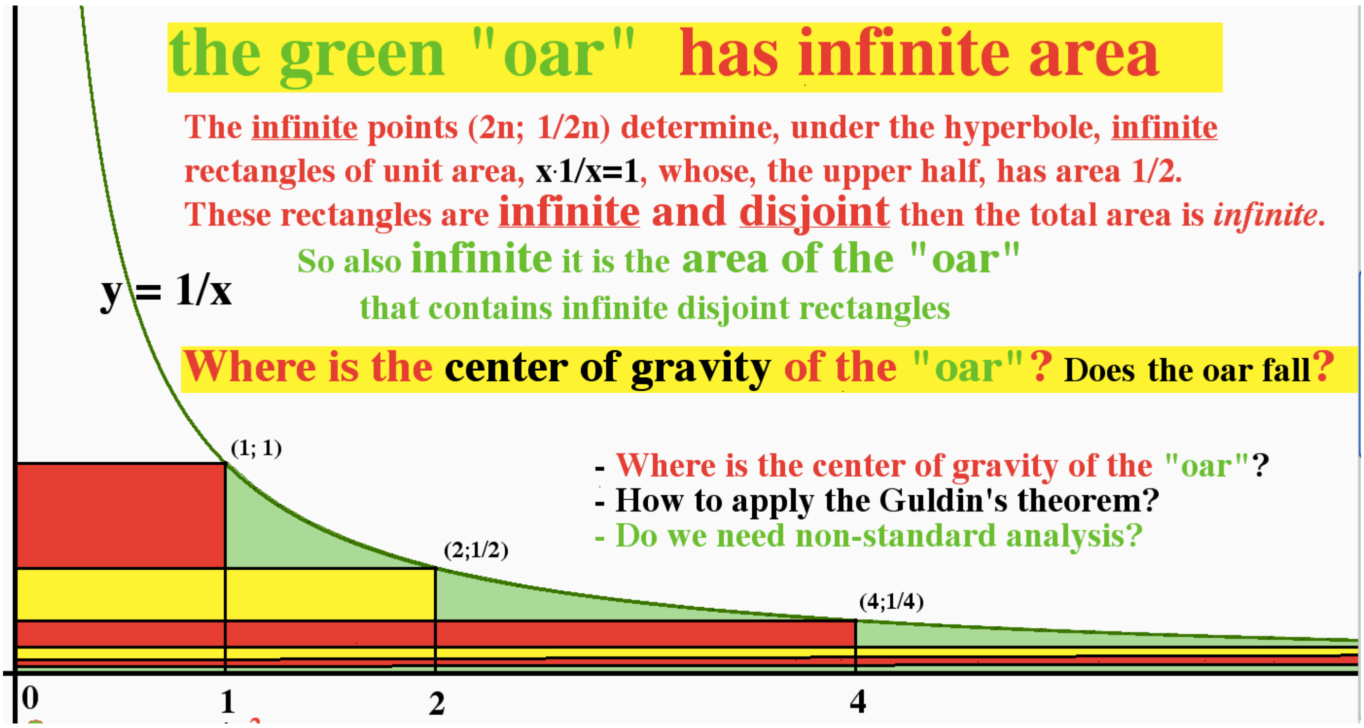
M1)

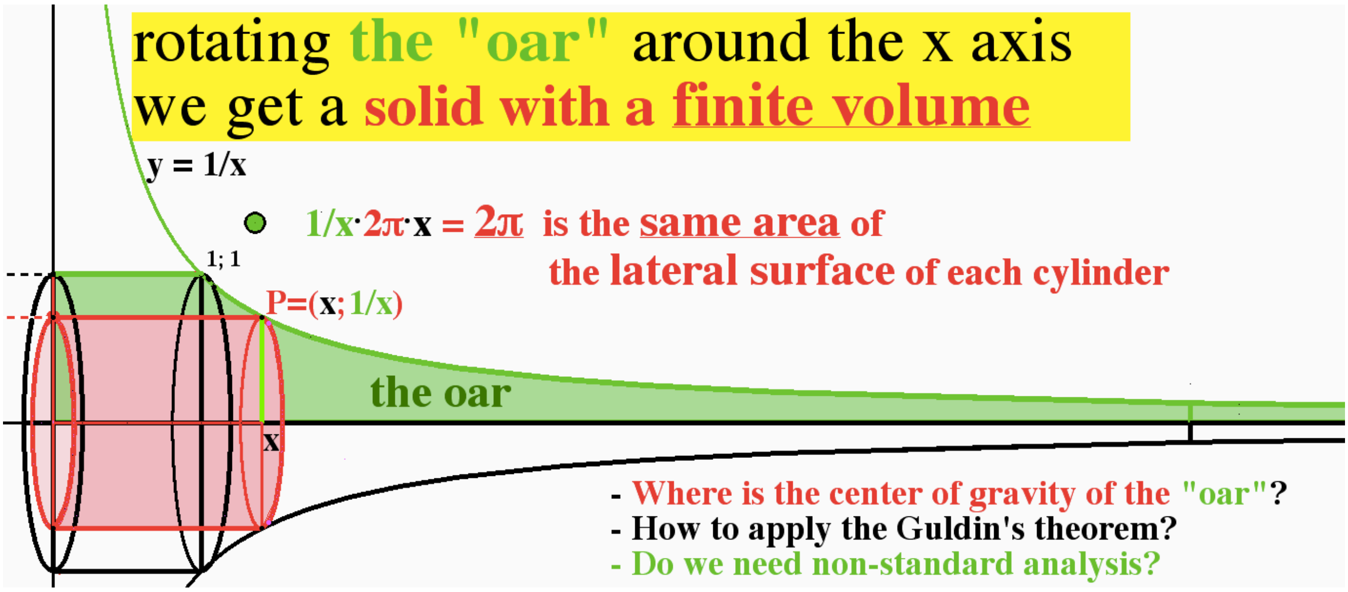


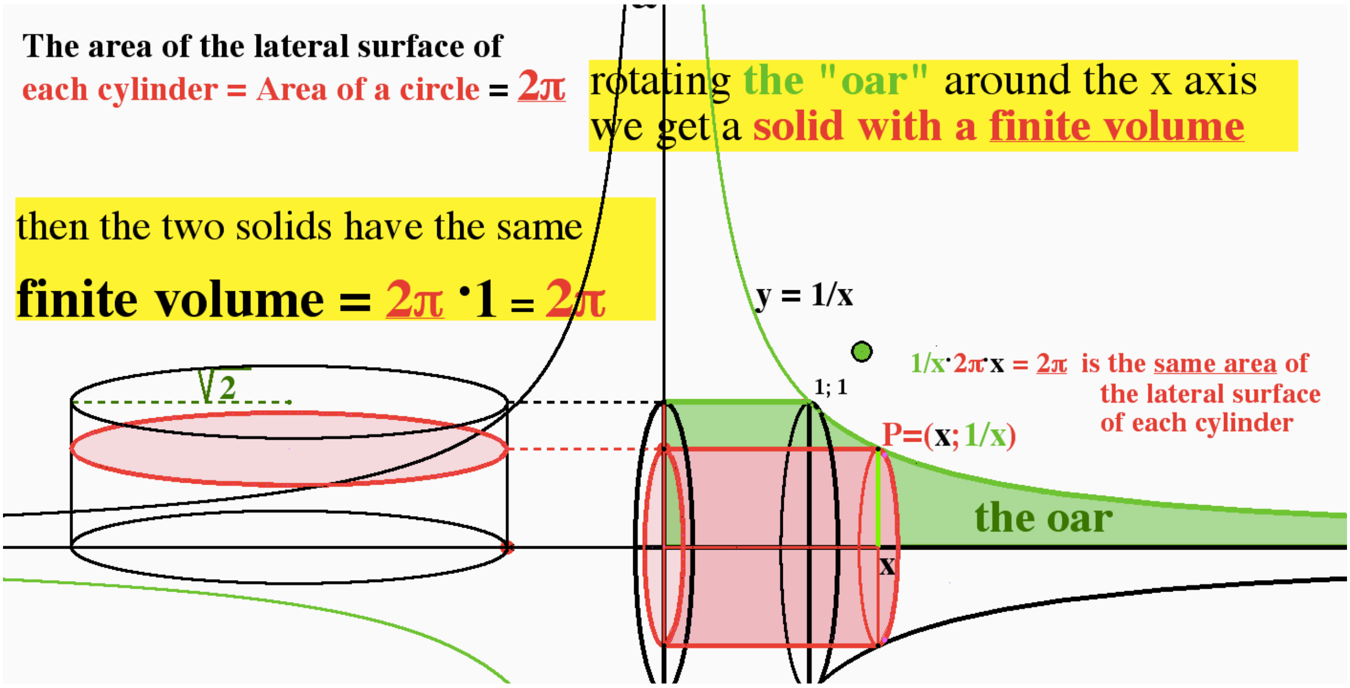


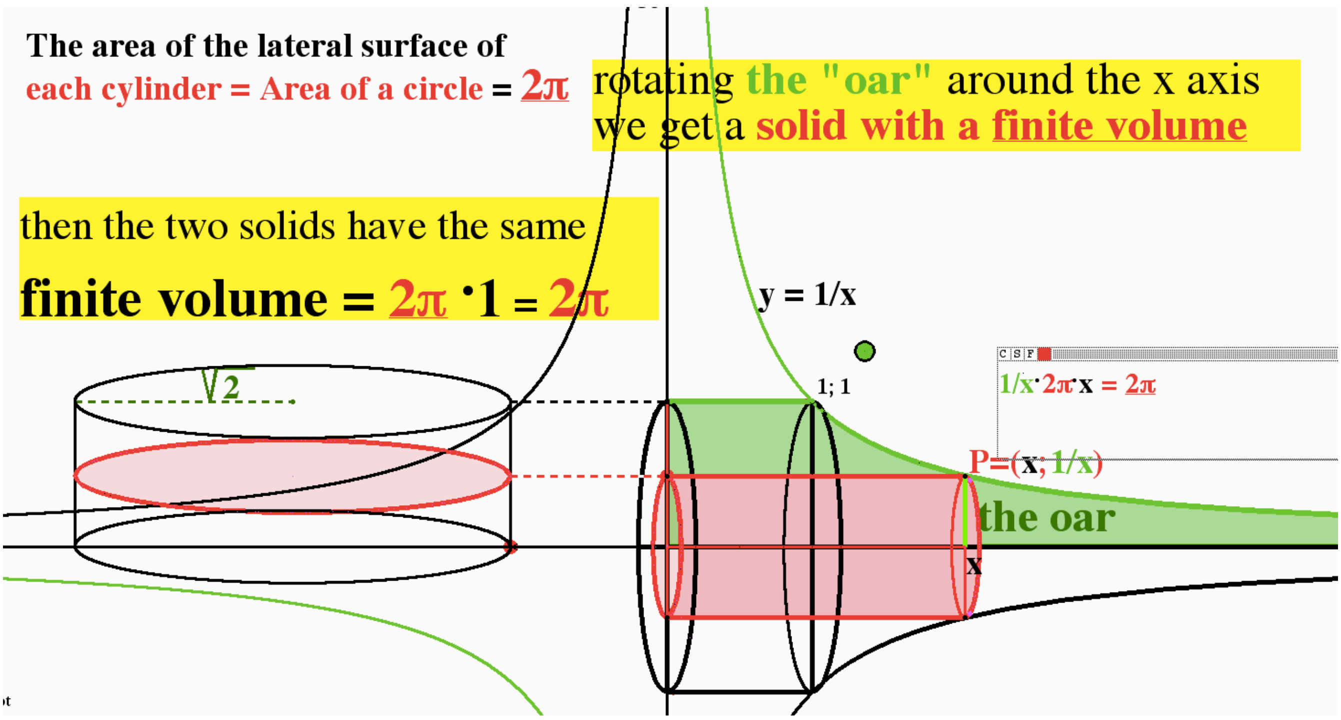
The fascination of the infinite,
knowing only the area of a rectangle:






rotating the "oar" around the x axis
we get a solid with a finite volume
y = 1/x
1/x·2π·x = 2π is the same area of
the lateral surface of each cylinder
1; 1
P=(x; 1/x)
the oar
x
- Where is the center of gravity of the "oar"?
- How to apply the Guldin's theorem?
- Do we need non-standard analysis?


The area of the lateral surface of
each cylinder = Area of a circle = 2π
rotating the "oar" around the x axis
we get a solid with a finite volume
then the two solids have the same
finite volume = 2π ·1 = 2π
y = 1/x
√2
1/x·2π·x = 2π is the same area of
the lateral surface
of each cylinder
1; 1
P=(x; 1/x)
the oar
x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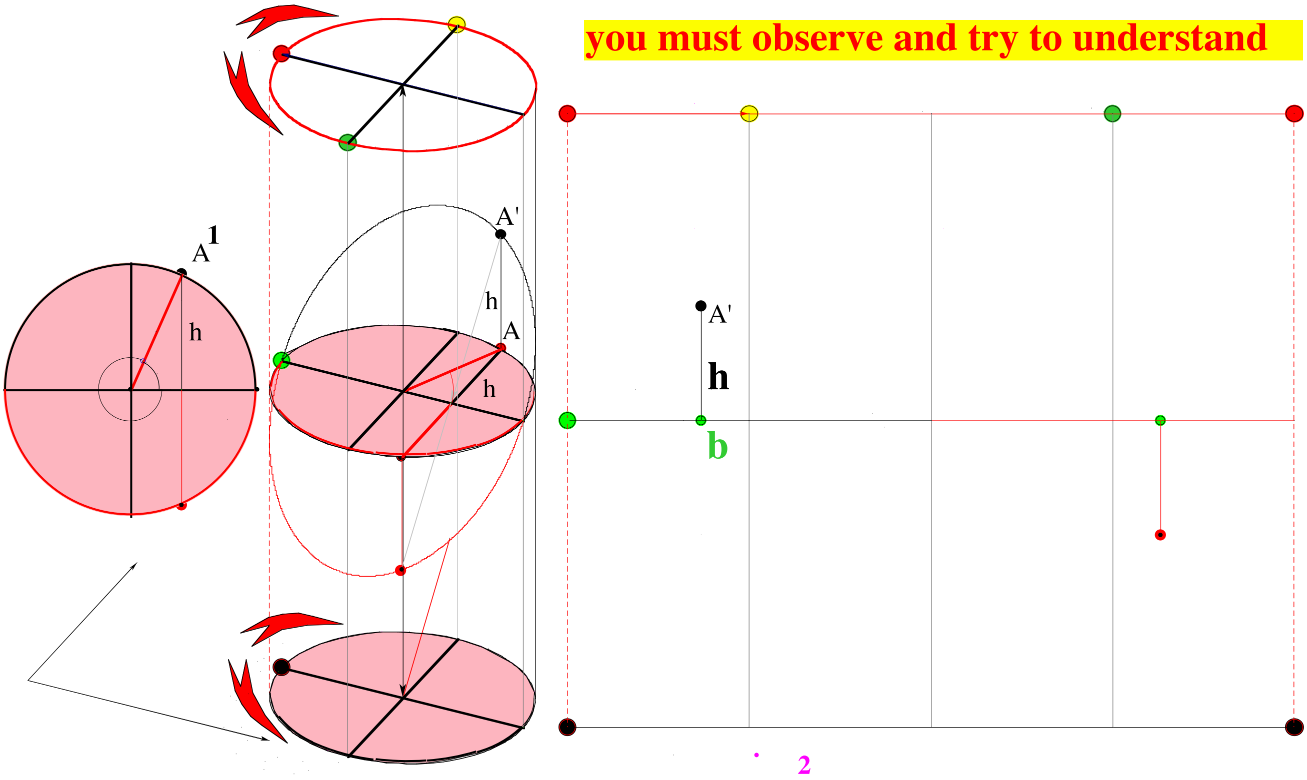
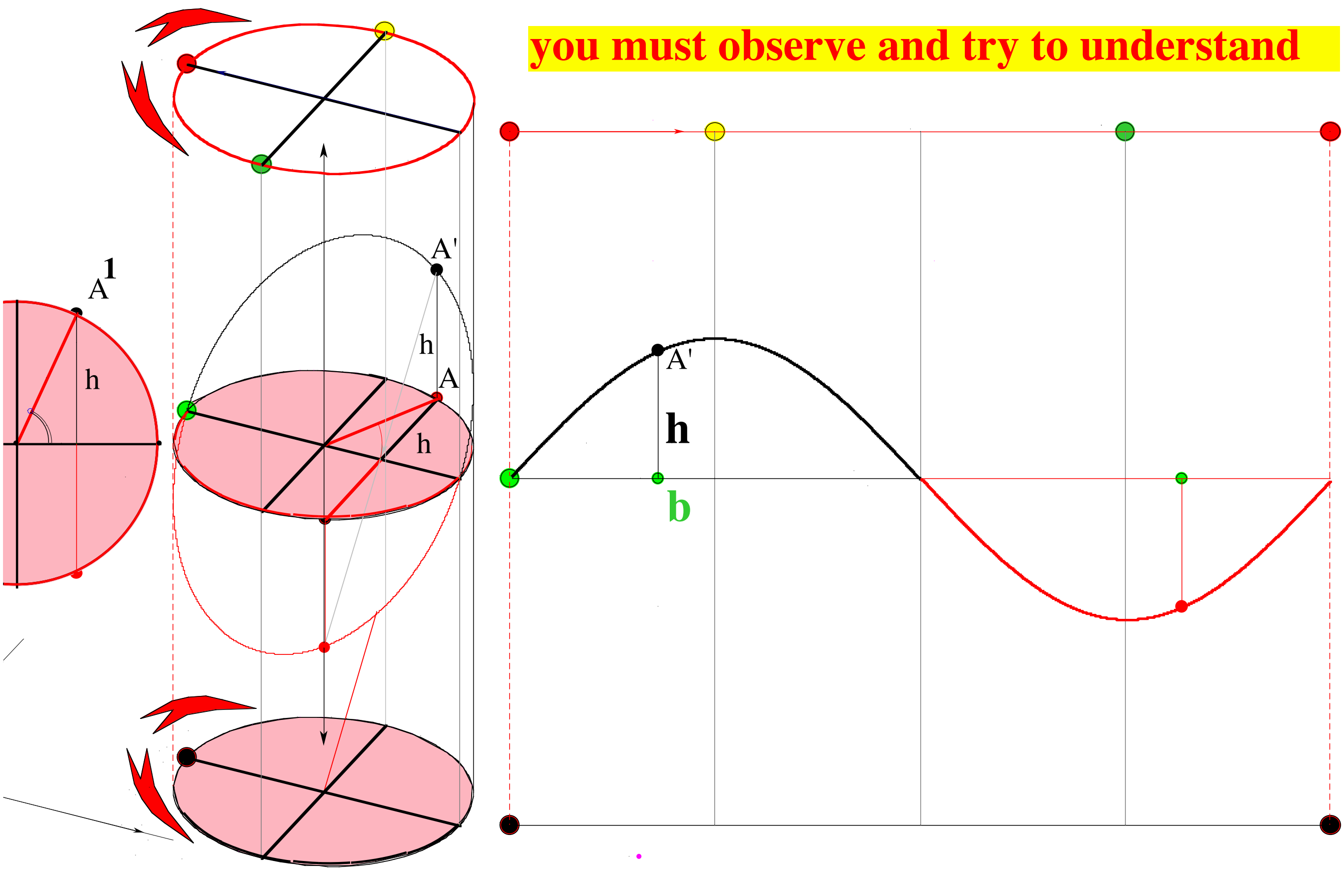
M4)

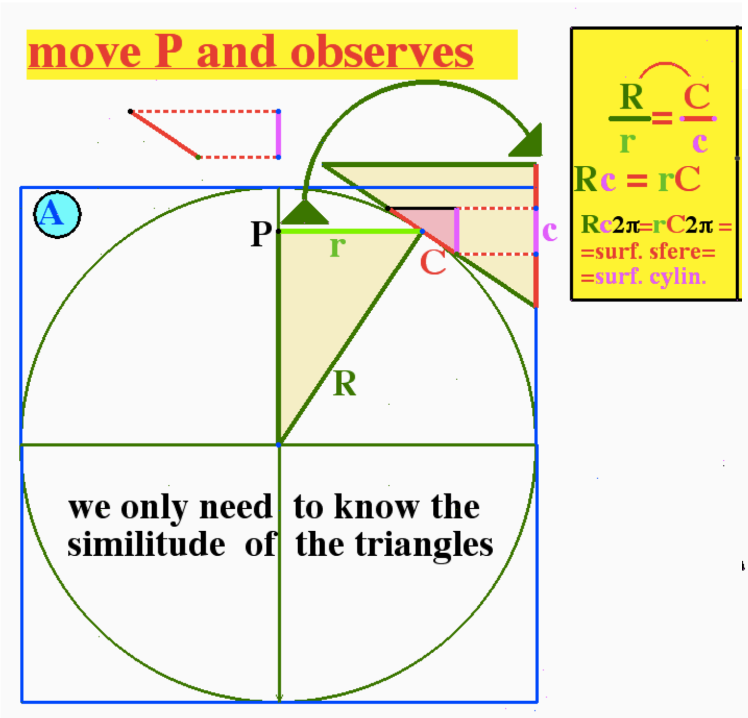


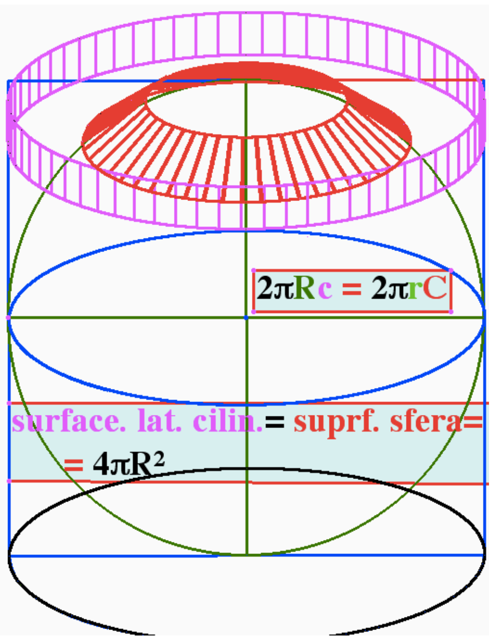


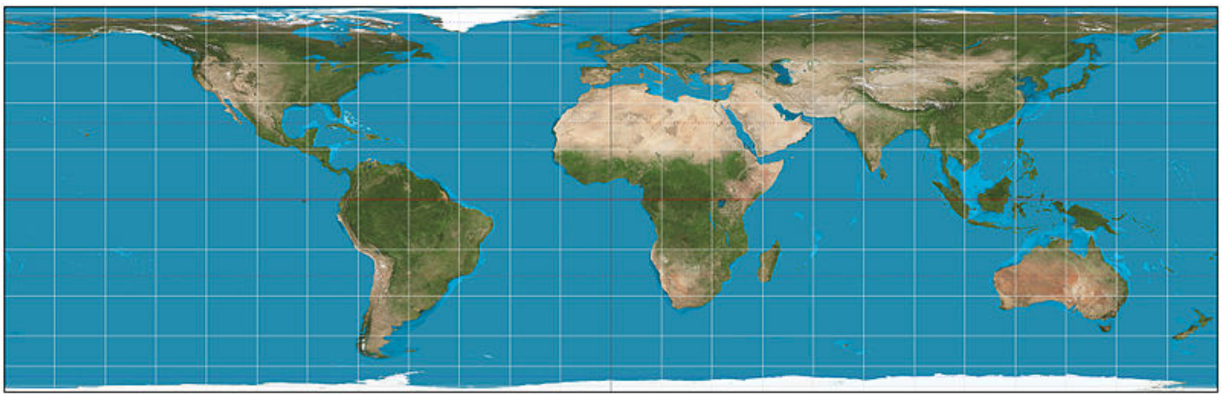
With movement you can better understand

Geometry and cartographic projections

Students can discover the cartographic properties

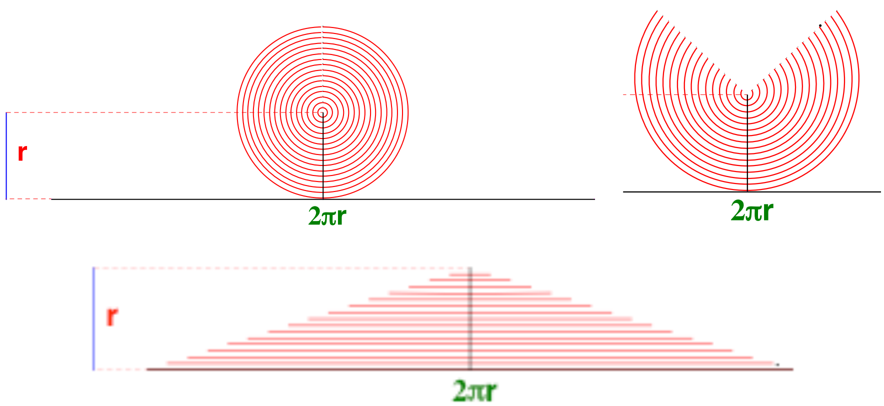




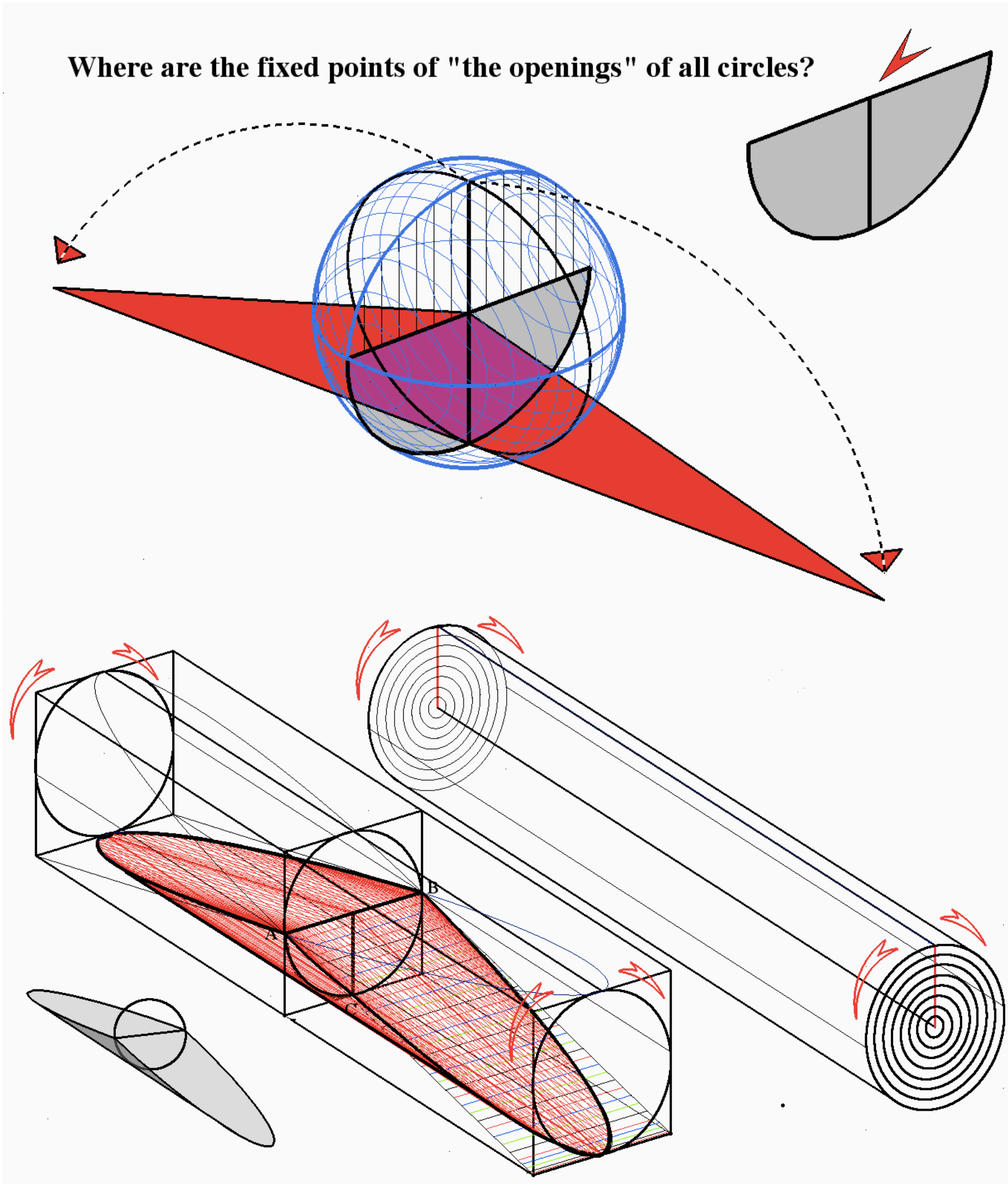


The Lambert cylindrical equal-area projection (Equivalent)

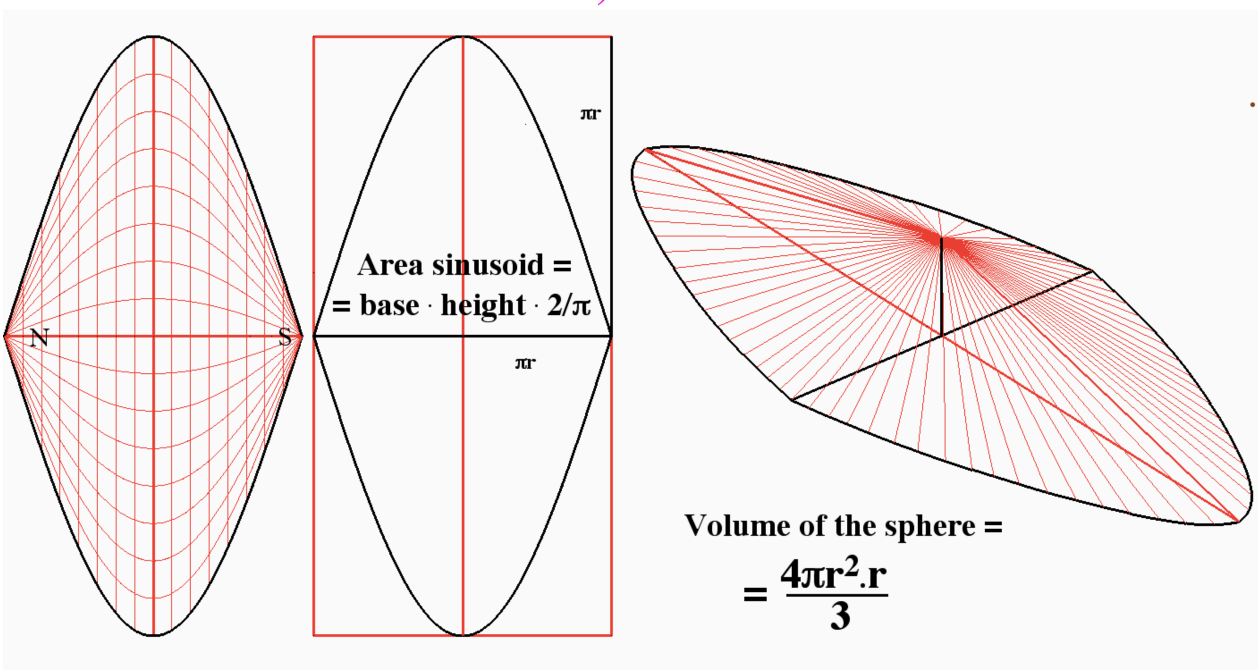
M2)



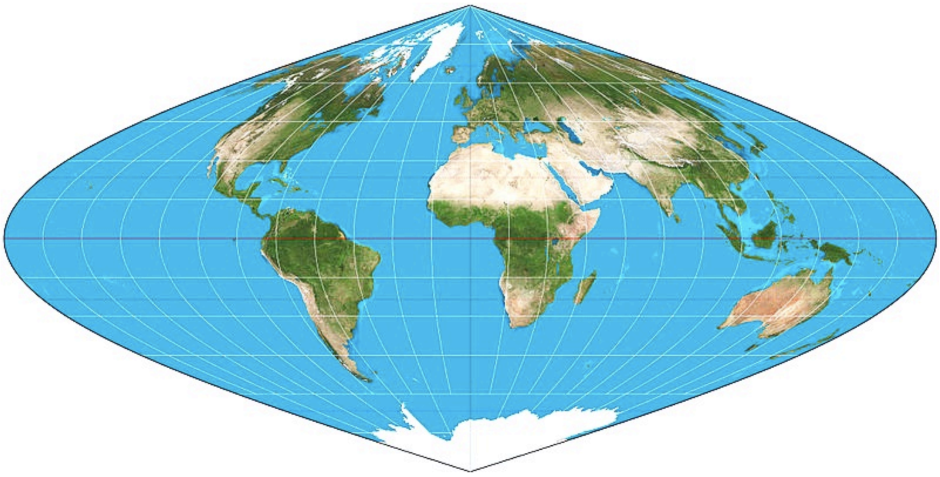
The fixed points are on the black segment


Where are the fixed points of "the openings" of all circles?
A
B

M4)


N
S
πr
πr
Area sinusoid =
= base · height · 2/π
Volume of the sphere =
= 4πr².r / 3

Sinusoidal cartographic projection
(Equivalent)

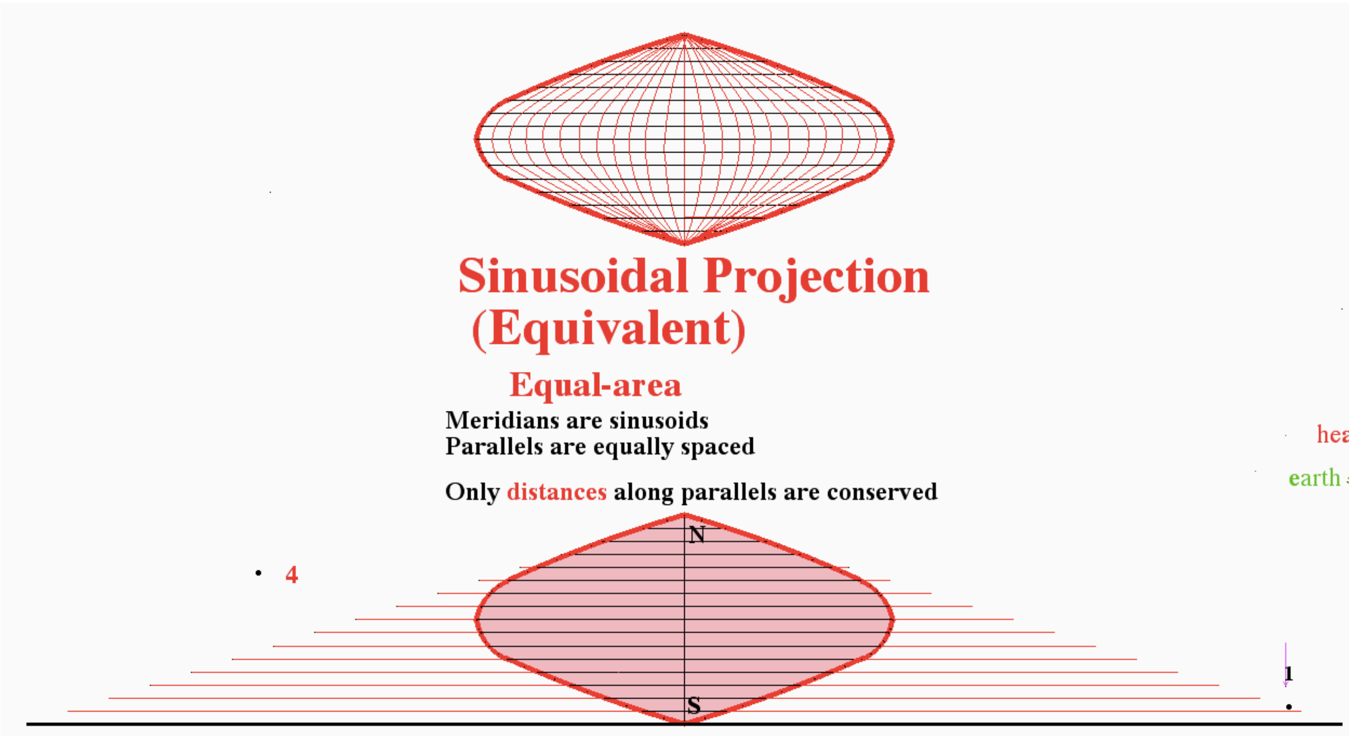


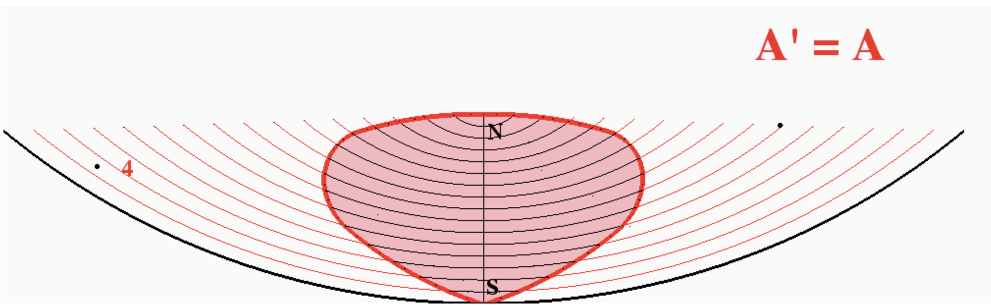
Equal-area

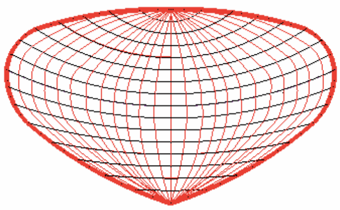
Meridians are sinusoids

Parallels are equally spaced

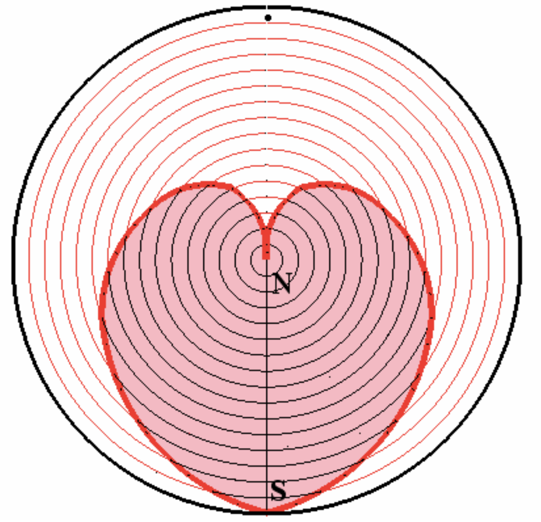
Distances along parallels are conserved

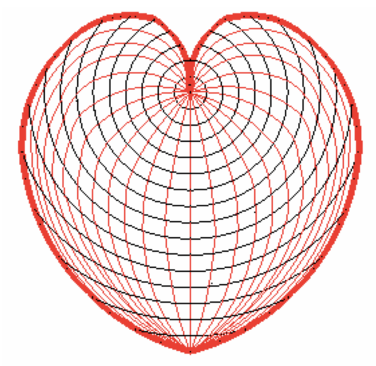






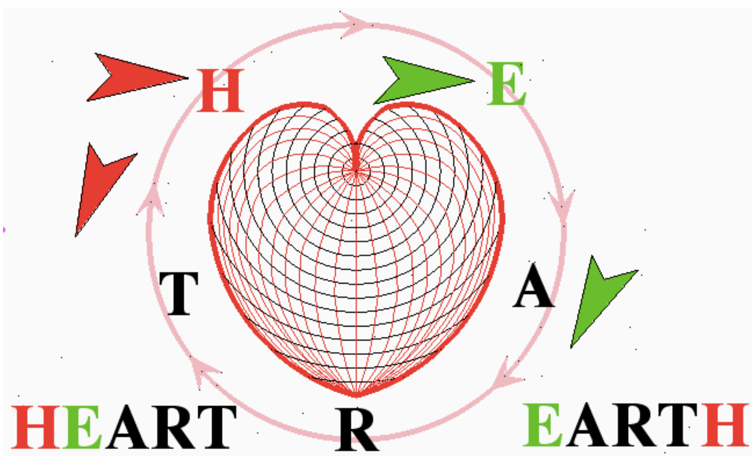
A' = A



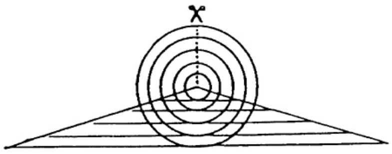


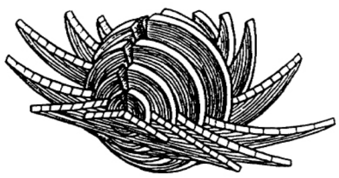
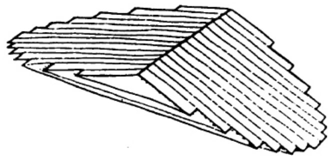
Werner Cordiform Projection
(Equivalent)

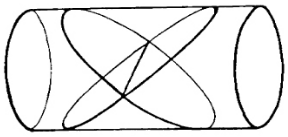
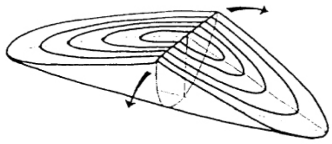
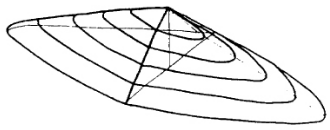
1) Equal-area
2)! Distances from the North Pole are conserved
3) Curved distances along parallels are conserved
4) Distances along one meridian are conserved

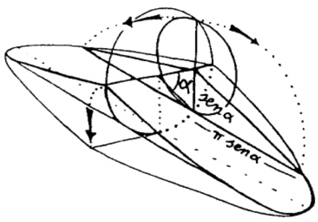


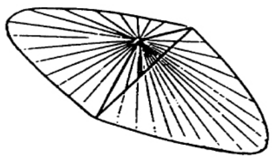
Summary

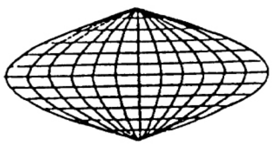
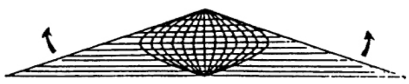
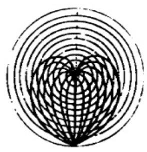







[91]

Are you an ecologist?

I hope that the butterflies will live forever.

(91) Hear the art

Mario Barra