

CIEAEM67 (2015) Discussion Paper

Conference Theme: Teaching and learning mathematics: resources and obstacles

Introduction

Teaching and learning mathematics is a complex system, involving a plurality of factors and components, ranging from the epistemology of the discipline to cognitive psychology, socio-cultural environments, affective elements, and technological devices. At the very core of the system, making sense in doing mathematics is widely considered as a basic requisite for constructing knowledge. In this regard, it is worth analyzing mutual relationships between real objects and mathematical constructions, the role of thinking processes and languages (often related to embodied experiences), and the influence of beliefs and emotions. All factors can be double-faced, i.e., they can provide resources and/or obstacles for the development of mathematical knowledge. In this regard, the professional expertise of the teacher is of crucial importance: in fact the teacher is responsible for being up to date not only about the content aspects of the discipline, but also about those factors that interact (and interfere) with the teaching-learning processes. It is necessary for the mathematics teacher to be aware of these issues, both in designing classroom activities and in managing them with the students.]

The four subthemes (and related questions) we propose in the following are to be considered as a means to promote investigation and facilitate discussion. All the subthemes are closely interrelated: their distinction is purely functional to assist the organization of the working groups during the conference.

Subtheme 1. Mathematical content and curriculum development

The relationship between mathematics as a discipline and the mathematical content to be taught reminds us of the dialectic between theory and practice, which has received increasing emphasis since the 1990s (see, e.g., Brown & Cooney, 1991; Burton, 1991; Godino & Batanero, 1997; Wittmann, 1991). In the search of boundary conditions to mediate knowledge between the two poles, there is evidence that any conception which assigns to "theory" the role of instructing "practice" is doomed to fail and, consequently, there is a growing need for developing interaction between the two poles, and for co-operation between the actors involved in the education system (Bartolini Bussi & Bazzini, 2003).

Since the 1980s, an important contribution in the debate was given by Chevallard, who studied the didactical transposition phenomena, producing elements of knowledge about didactical systems and the content for mathematics teaching. This led to the development of the theory of didactic transposition as well as its practical realization (Chevallard, 1985). This idea has been further developed, in the 1990s and beyond, into a more general study within which mathematics is practised in terms of different praxeologies (combining praxis and logos).

Focusing on the epistemology of mathematics, and noticing persistent students' difficulties related to specific concepts, Brousseau (1997) discussed the notion of *epistemological obstacle* in mathematics. This idea has inspired research in mathematics education, opening the way to the search for other kinds of obstacles, related to didactical and cognitive aspects, as well as critique of the idea of epistemological obstacle, on the basis of historical-cultural discussion (Radford, 1997).

The dialectical interaction between theory and practice grounds the work of curriculum developers, mainly when different actors (researchers, teachers, school managers) are asked to work together. In

such cases, curriculum development can be a great opportunity for co-operation and mutual enrichment, and make a positive contribution to the school (Bazzini, 1991). This theme will be also discussed in Subtheme 2 (see below).

The choice of content to be included in the curriculum is an important issue requiring attentive investigation in any context. Along with traditional topics, such as arithmetic, algebra, and geometry, relatively new topics need to be included in the curricula: Probabilistic and stochastic thinking constitute one striking example.

In recent years, most countries have introduced or developed statistical content in primary and secondary mathematics. The reasons are many: taking into account the rise of stochastic power in the discipline of mathematics, the will to develop other teaching approaches based on modelling from real situations, and interdisciplinarity, as a societal demand.

In Higher Education, more and more courses are incorporating statistics at the Bachelor level as in Doctoral programs. At this level, the sectorial variations are multiple (statistics for biology, management, psychology, etc.) with, as noted by Jeanne Fine (2010), in the words of Bourdieu, a high risk of hyperspecialisation and a weakening of the identity of the discipline. The foundations are supposedly acquired during previous schooling, and teaching of statistics is reduced to the presentation by non-specialists implementing techniques using specialized software. The operational dimension of knowledge is privileged at the expense of systematic and historical dimensions (see Fabre 2010), with the risk that students do not master basic statistical concepts, as highlighted in numerous research studies (see, in particular, Batanero et al, 1994; Delmas et al, 2007). The multiple epistemologies, most often not clarified, are a source of difficulty for students who do not identify where the professor or teacher is coming from, epistemologically (Armatte 2010).

It is true that statistics is a discipline whose epistemology is complex. However, it is important that this discipline is taught by specialists in higher education and is integrated into mathematics lessons in secondary school such that it is not diluted in the host disciplines (Gattuso, 2011). But there are many differences with mathematics, differences which must be made explicit in the context of the training of mathematics teachers. In statistics, students should be led to give up their deterministic worldview and to consider the lack of certainty as a feature of reality (Meletiou-Mavrotheris & Lee, 2002). A fundamental difference between statistics and mathematics is that, in statistics, the context has a special status: it is an integral part of the problem. The risk of misunderstanding between teacher and students, linked to the different representations, then becomes greater (Hahn, 2014). In statistics students should jointly master inductive and deductive reasoning (Fine, 2010), and combine the two perspectives: the data-centric approach and the more formal modeling (Armatte, 2010; Peters, 2011). This is not only to master the concepts but also to develop a statistical way of thinking (Gattuso, 2011), integrating the use of technology, which is essential in Statistics (Serrado et al, 2014).

The previous discussion opens the way for contributions to the subtheme 1 of the CIEAEM67 Conference, which focuses on issues related to the epistemological aspects of mathematics relevant to educational aims, and frames them in terms of the obstacle/resource dialectic. Subtheme 1 will focus on the following questions:

- *Which obstacles may interfere with teaching? What is their nature? What could be possible strategies to avoid/overcome them?*
- *Which obstacles interfere with learning? What is their nature? What could be possible strategies to avoid/overcome them?*
- *What are the resources and obstacles in different national curricula?*

- *What professional expertise is needed for developing and implementing curriculum?*
- *Is there any specific content in need of special attention?*
- *Should statistics be introduced in the primary school? How should we think about the preparation of teachers who will teach statistics at each level (primary, secondary, higher education)? What are the differences/complementarities between mathematics and statistics?*

Subtheme 2. Teacher education

Mathematics teacher education has been receiving increasing attention in research over the last decade (Clark-Wilson et al., 2014; Even & Ball (Eds.), 2009; Wood (Ed.), 2008). This ‘emerging field’ (Adler et al., 2005) has its roots in previous research on classroom teaching-learning processes. With the progressive diffusion of new learning and teaching models since the 1960s, the role of the teacher in the classroom has changed radically. In fact, new approaches to learning also require new approaches to teaching: this change is not spontaneous; on the contrary, in order to take place it needs to be fostered by suitable teacher education initiatives.

Research has pointed out different aspects with respect to mathematics teacher education: from the specificities of the knowledge needed by teachers to affective factors, from the inclusion of new technologies to systemic analyses.

Reflection on the kind of knowledge that characterises the mathematics teacher in his/her professional work has been carried out in the seminal work of Shulman (1986). Ponte et al. (1994) support the idea of blending mathematical content with pedagogical knowledge, drawing on different components of current knowledge to produce a restructuring of the teacher's craft knowledge. This pedagogical content knowledge has a much broader scope than just the representation of the subject matter: it must include "a comprehensive body of images, principles, and rules for action, some general, some more specific, organized with a clear rationale, bearing on the specific nature of the underlying content and powerful enough to guide the action of the teacher" (p. 358). Steinbring (1998) explores a specific component of professional knowledge for mathematics teachers, namely “epistemological knowledge of mathematics in social learning settings (p. 160)”. He claims that “teachers surely need mathematical content knowledge and pedagogical knowledge; and, within the domain of pedagogical content knowledge, they also need epistemological knowledge, so that they are able to assess the epistemological constraints of mathematical knowledge in different social settings of teaching, learning, and communicating mathematics. This important component of epistemological knowledge of mathematics in social learning settings is not a systematized, canonical knowledge corpus, which could be taught to future teachers by way of a fixed curriculum. Rather, the epistemological knowledge consists of exemplary knowledge elements, as it refers to case studies of analysis of teaching episodes or of interviews with students, and comprises historical, philosophical, and epistemological conceptual ideas” (p. 160). Ball and Bass (2003) frame the typical features of mathematics that are involved in teaching within the Mathematical Knowledge for Teachers model, identifying the Specialized Content Knowledge as an important sub-domain of mathematical knowledge, strictly connected to the work of teaching. Specialized content knowledge intertwines often with knowledge and competences related to digital technologies, which have also gained increasing relevance in the teacher education context (Bairral & Powell, 2013; Drijvers et al., 2010).

On the other hand, several studies have investigated the social aspects of teacher education programs, especially the involvement of teachers in joint analysis and reflection together with researchers. Within the research literature we find important notions such as *community of practice* (Wenger, 1998) and *communities of inquiry* (Jaworski, 2006); the cornerstone of these studies being

the notion of critical reflection, conceived not only as a fundamental attitude to be developed by teachers, but also as a professional responsibility. This idea is strictly interrelated with that of joint collaboration between teachers and researchers, as Krainer (2011) stresses when he suggests looking at researchers as “key stakeholders in practice” and teachers as “key stakeholders in research.”

Besides epistemological and social dimensions, the affective dimension comes to play an important role in teacher work and in teacher education as well. It includes studying the influence of teachers’ beliefs and emotions on their mathematics teaching. In fact, as Zembylas (2005) underlines:

teacher knowledge is located in ‘the lived lives of teachers, in the values, beliefs, and deep convictions enacted in practice, in the social context that encloses such practices, and in the social relationships that enliven the teaching and learning encounter’. These values, beliefs and emotions come into play as teachers make decisions, act and reflect on the different purposes, methods and meanings of teaching. (p. 467)

This is particularly relevant, especially concerning primary teachers, who are generalist teachers and sometimes have to teach mathematics despite their personal dispositions towards mathematics. Hence, teachers’ beliefs and emotions towards mathematics can constitute obstacles to effective teaching practice. The study of the conditions under which this hypothesis is true remains an open problem. On the other hand, personal negative experiences and emotions may also become resources for teachers, as suggested by Coppola et al. (2013), focusing in particular on future teachers.

Finally, mathematics teacher education processes also need to be considered from a systemic point of view, with a focus on the relationships and dynamics between the several “variables” included in such complex processes as: teachers’ knowledge and practices, results from research, institutional constraints (national curricula in particular), traditions, cultural aspects, and so on. Considering this complexity, teachers’ development can be considered as a meta-didactical transposition process evolving over time (Arzarello et al., 2014).

Starting from this discussion, and from the contributions of the accepted papers, subtheme 2 in CIEAEM67 aims at rethinking the complexity of teacher education in terms of resources and obstacles for teaching and learning mathematics. The following questions may further guide the discussion:

- *How is it possible to support teachers to develop suitable knowledge and competences in digital technologies, so that they are effective in their mathematics teaching?*
- *What are the main obstacles for mathematics teacher development?*
- *How can the social dimension become a resource for teacher education? What are the challenges of programs strongly based on social interaction in communities of practice/enquiry?*
- *How can the affective dimension become a resource for teacher education?*

Subtheme 3. Classroom practices and other learning spaces

Mathematical thinking arises and develops in a complex interplay of languages and representations, through reference to intuitions, metaphors, and analogies, and by making use of various artefacts and tools, which interact with our bodily nature. All these components are crucial for teaching and learning activities within the classroom context, as well as within other learning spaces: in light of the Conference theme, they can constitute possible resources or, on the contrary, obstacles for the mathematics learning.

Whereas there has been a focus on language and written representations since the 1980s, more recently attention has also been given to embodied forms of representation and thinking, such as gestures, considered mainly as resources for teaching and for learning (Arzarello, 2008; Arzarello et al., 2009; Radford, 2002, 2014). Other studies have investigated the role of new technologies and ICT as possible mediators for learning (Drijvers et al., 2010). Thus, Subtheme 3 includes the discussion on the possible uses of new technologies as resources for the learning of mathematics, but also on the possible obstacles that the introduction of new technologies could produce at several levels (cognitive, didactic, communicative, etc.).

Concerning classroom practices, the role of the teacher comes to the fore. Even from possibly different theoretical positions, the teacher is usually intended as a resource for students' learning. In this regard, teachers need to deal with different cognitive demands, in particular with those of students having learning difficulties in mathematics, as widely discussed in literature (Dehaene, 1997, Landy & Goldstone, 2010). A conscious use of specific teaching strategies suitable for students diagnosed with learning disorders, in particular with developmental dyscalculia (Butterworth, 2005; Dehaene, 1997), is also important for those students who are not officially diagnosed, but have learning difficulty profiles very similar to those of dyscalculic students. Therefore, the development of innovative teaching support looks like an ever more necessary goal for research in mathematics education in general, and for teachers in particular.

Although school is the most important institution for learning, we know that it is not the only place where we learn. But, what do we mean by learning? It is common to find teachers with a restricted view concerning what it means to learn mathematics. Often learning is associated with the reproduction of counting procedures and calculation formulas. Although this idea has been overtaken, at least for research within mathematics education, it seems, unfortunately, that some teaching or training practices are still restrictive and do not acknowledge that learning can be observed through different lenses. We learn in formal and non-formal spaces (museums, distance learning programs, game playing, etc.), in face-to-face or online dynamic environments. We believe that teaching mathematics in any context should promote the development of thinking that offers potential for the student in their present and their future, regardless of their of future occupations or professional work. Processes such as developing curiosity, critical thinking, reasoning, and motivation to learn, as well as developing modes of verification, refutation, and deduction should all be leveraged both in the classroom and also in non-formal learning spaces.

Subtheme 3 includes the discussion about:

- *What are the features that characterize the teacher's practices as resources for students and how is it possible to foster these features?*
- *A provocative question: Can a teacher be an obstacle to the students' learning? Why and how does it happen? How could it be prevented?*
- *How can technologies and ICT be possible mediators for inclusive teaching and learning?*
- *How can embodied forms of representation and thinking, such as gestures, or other different registers of representation, such as visual-verbal, visual-non-verbal, auditory, and kinaesthetic, be considered as resources for inclusive teaching and learning ?*
- *Which resources or teaching strategies are being used to enhance the learning potential of all students, particularly those with learning difficulties?*
- *Which new aspects of mathematics learning can be improved in formal learning spaces or in non-formal environments?*

- *What the advantages or restrictions of ICT or more conventional resources (e.g., the manipulative ones) in promoting mathematical learning within formal or informal contexts?*

Subtheme 4. Cultural, political, and social issues

Since the 1980s at least, there have been challenges to assumptions that mathematics is culture- and value-free (Bishop, 1988; D'Ambrosio, 1985; Ellerton & Clements, 1989). There is also a developing awareness that mathematics education itself was not only portrayed as culture- and value-free, but also was effectively excluding or alienating many girls and women as well as boys and men who did not conform to the stereotypes found in classroom and textbook examples, or the choices of abstract, highly theoretical curricula. To epitomize this shift of research in mathematics education, the terms 'social turn' and 'sociopolitical turn' (Gutiérrez, 2010; Lerman, 2000) have appeared. Now, it has become broadly accepted that we can no longer think of mathematics and mathematics education as far removed from cultural, social and political issues when studying and trying to improve mathematics education.

Cultural, political, and social contexts can be considered as obstacles and/or as resources for students' success in mathematics. On the one hand, we can consider these as obstacles for students' access to, and their achievement in, mathematics education. Although less prevalent in Western countries, but nevertheless of fundamental importance, the physical access to schooling and mathematics classrooms has received attention (e.g., Kazima & Mussa, 2011). On a second level, curricular reforms and counter-reforms have often transformed the obstacles for mathematics learning that some social groups face (e.g., Jablonka & Gellert, 2011; Vithal & Skovsmose, 1997). This second level is concerned with the distribution of different forms of mathematical knowledge: Who gets access to which forms of mathematical knowledge? On a third level, the question has been raised as to how instructional and educational strategies complicate or impede access to, and participation in, institutionally and socially valued forms of mathematical activities for particular groups of students (e.g., Straehler-Pohl et al., 2014). Cultural (e.g., the culture-specific importance of orality), political (e.g., policies for integration of migrants), and social (e.g., relative poverty) conditions, taken separately, but mostly combined, often translate into obstacles for the teaching and learning of mathematics.

On the other hand, cultural, political, and social conditions can be regarded as resources. This is quite obvious in the case of privilege, where students' backgrounds and foregrounds easily prove beneficial for the acquisition of the school subjects' dominant registers and orientations to meaning (e.g., cultural capital and middle-class codes). The crucial point is if, and if so, how, not-yet-valued experiences and activities of underprivileged students can be used as resources for the teaching and learning of mathematics. As an example, Barton and Frank (2001) reflecting on minority cultures ask: "What are the conditions under which" (...) children, for whom the (conventionally) 'basic' mathematical concepts are not readily available because of incommensurable concepts powerfully present in their own cultural-linguistic heritage, "have a cognitive advantage in mathematics, and what is the nature of that advantage?" (p. 147). Healy and Powell (2013), examining multiple resources for mathematics learning, conclude that there is a wealth of studies showing how being multilingual relates positively to cognitive development. These studies also call for more invitation and encouragement of students to use their linguistic resources within mathematical activities. Bringing these two perspectives together, understanding the cultural, political, and social conditions that create obstacles for mathematics teaching and learning, might lead us to understand the micro- and the macro-social processes that disadvantage individuals.

As a matter of fact, diversity is an essential part of what it is to be human. Even within the same family unit there are differences between the children in terms of their interests and aptitudes. Within classrooms where students apparently share the same social and cultural backgrounds there

is no uniformity. Particularly in recent times of global flows of people, many classrooms are likely to comprise students with diverse social, cultural, and linguistic backgrounds, and these offer both a resource and a challenge to teachers who may lack systemic support, as well as being expected to work under increasing pressures of time and accountability. This is in the face of mission statements and policy documents that state that each child or learner is an individual and should receive personalised attention from his/her provider of education.

Finally, understanding how cultural, political, and social conditions can become resources for learners might require us to analyse how curriculum, teaching strategies and learning scenarios can be more finely tuned to the backgrounds and foregrounds of particular groups of students.

Questions:

- *How do cultural, political, and social contexts restrict access to, and participation in, valuable forms of learning mathematics? How can these restrictions be overcome?*
- *How can underprivileged students' backgrounds and foregrounds be used as resources for the teaching and learning of mathematics?*
- *How could we rethink theories and practices of mathematics teaching to improve cognitive and affective outcomes for bilingual/multilingual students?*
- *How could we foster the inclusion of students from different cultural backgrounds within the mathematics classroom and in the broader society?*
- *How could we deal with challenges of gender stereotypes and other gender-related issues and the inequalities they create?*
- *How do policy designers take into consideration any kind of diversity and inequality in your country or region (e.g., the EU)?*

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