COMISSION INTERNATIONALE POUR L'ÉTUDE ET L'AMÉLIORATION DE L' ENSEGNEMENT DES MATEMATIQUES
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Discussion paper

Mathematics and realities

The theme of this conference refers to fundamental questions about mathematics, its existence, its discovery, and its relations with other sciences, but also issues related to its teaching and learning in the twenty-first century.

The PISA 2012 assesment and analytical framework (PISA, 2012) offers a definition of mathematics based on mathematical relations with the "real world":

Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (PISA, 2012)

The reality of mathematical objects can be seen only through an act of representation. And even if one believes Freudenthal, if "our mathematical concepts, structures, ideas have been invented as tools to organize the phenomena of the physical, social and mental world" (Freudenthal, 1983, p ix), relationships between mathematics and realities, as well as relationships of mathematics with other sciences, come up against and face problems of translation due to the nature of mathematical objects themselves.

Thus, the relationship of mathematics to realities, being philosophical, social, societal or educational, raises questions that the conference will address through its sub-themes of:

- Mathematics in relation to other disciplines, by questioning the relationship, at a school level and at an academic level, between disciplines and addressing the question of the reality of objects and concepts that depend on the appropriate epistemology of each discipline.
- Logic in mathematical practices, addressing issues of logic as being part of mathematics and mathematical discovery, as well as the links between logic and reasoning through questions such as: What is the role of logical reasoning? What are the links with argument, evidence? What teaching can promote the acquisition of reusable logic skills?
- Technology and mathematics experiences, sub-themes in which the mathematical experience will be questioned: Is mathematics used by students to solve a mathematical task in a paper-and-pencil environment different from that he or she would use in a technological context?
- Multiculturalism and reality: Teaching realities in multicultural classrooms with multiple cultures can equally be addressed from the perspective of professional cultures, social cultures, ethnic cultures, etc.

Sub-theme 1

Mathematics and its teaching in relation to other disciplines

In the school curriculum, mathematics is the object of education fully in its own right, but it is also a tool for the teaching of other subjects (or matter). In addition, the performance of students in mathematics plays an important role in enabling them to further their educational studies.

The distribution of class schedules between school subjects identifies certain ideological and pedagogical predetermined positions on the value or utility of each discipline. The important role played by mathematics in the curriculum, and the significant private tutoring, suggest that this discipline creates a strong social and academic consensus on its value and usefulness.

Thus, many researchers have tried to find the best place to improve the teaching of mathematics and / or other disciplines as part of a re-prioritization of the order of the spatio-temporal organization of courses in relation to their content and their epistemological value. In this sense Lenoir (1993, 1994) speaks of school interdisciplinarity. This interdisciplinarity works at teaching and curricular levels, and pedagogical interdisciplinarity is a result of prior interdisciplinary work being done at both levels. Even within academic interdisciplinarity, the risk of simplification, among other things related to the predominant empirical concern (and certainly legitimate for teachers, in order to save time and energy) as well as from ideological positions - prioritization of materials, for example , which strongly influence the conduct of primary school teachers (Lenoir, 1992) - have often led to considering the implementation of interdisciplinarity essentially in terms of pedagogical action, then forgetting that it is not independent of the teaching work and curricular structure.

However, the organization of courses in the school does not foster for students the organisation of knowledge into a coherent system for dealing with complexity. There is not enough interest in the relationship between disciplinary learning, and it is limited to the acquisition of knowledge at the technical and methodological levels, aimed at passing from one course to the next, or from one discipline to another .

In this context, the question that is often asked and repeated in various forms is "What use is mathematics?" This issue has generated a large field of research and studies in mathematics education on the relationship and impact of mathematical modeling and the development of other disciplines.

Moreover, Legendre (1993) points out that from the scientific level, where the interdisciplinary question was first developed, various ways of categorizing and prioritizing relationships between scientific disciplines have been produced from the identification of methodological links, such as languages of mathematics or cybernetics, theories such as psychogenetic study of the development of thought or systems theory, or, alternatively, from the objects themselves of the disciplines concerned.

At the Mathematics Education Research level, this issue followed from the famous question-invitation of Revuz (1980): "Is it possible to teach mathematics?" which led to the brilliance of Mathematics Education and has upgraded the importance of operational contact between mathematicians, teachers, psychologists, epistemologists and mathematics teachers at all levels, which is the core and the leitmotif of CIEAEM.

More recent research highlights a particular way of teaching sciences through different disciplines in the prospect of promoting cooperation between each discipline, each of them carrying viewpoints on both the studied objects and the methodologies, while keeping the specific features of the subject. (Prieur et al., 2011, Aldon et al., 2012).

The educational institution finds itself today facing the very difficult problem of re- legitimizing its cognitive authority, its social responsibility, and pedagogic competence or teaching skills. The school must rethink its organization and content of its teaching, trying to gain the respect and attention of students and society. A formal convergence of science and technology has been promoted in many curricula since 2000 associated with the development of skills (the 12 key skills in Europe, the American standards) and the current issue focuses on the how (Coquidé, 2008): How can the teacher grasp this convergence and how can research help to develop or inform this?

In order to answer the previous question we must avoid the antagonism between different disciplines and realize the importance and requirement of working in a complementary way, and therefore to teach our students to work cooperatively, democratically, and in an interdisciplinary way. Since 2000, the orientation of the curriculum of compulsory education has fallen within the development of scientific literacy for all (Robine, 2009), although, as pointed out by Van Haecht (1990), one of the main reasons for disciplinary curricula dominating in most schools and, thus, for integrated curricula existing only in a few schools, is as a result, at least partially, of the efforts to maximize the production of high status knowledge in the school system.

This great scientific and humanist project is joined by education research, taking as its object interdisciplinarity, including proposals for the space in which it is possible to construct it, taking into account the curriculum, the teaching practices that emerge, and the effects on student learning.

In this context, there is again the question of the usefulness of mathematics: How can mathematics interact with other disciplines to gain an understanding of a problem in several dimensions, seen as a complex phenomenon?

It may be a paradigm shift in science education that follows the change of the scientific landscape and its mixture with the human sciences, as well as new emerging disciplines such as bioinformatics, biophysics, ...

According to Le Moigne (2002) "... in these calls for interdisciplinarity and for research, education and human activities are expressed in two main streams, one focusing on methodological transfers from one discipline to another (known as "Pluri- ") , the other focusing on the socio-cultural legitimation of the knowledge produced and its producers (known as "Trans-") .

Moreover, according Resweber (2011), interdisciplinarity is part of a journey that involves upstream, the moment of the multidisciplinary and, downstream , the transdisciplinarity. Multi -, inter - and transdisciplinarity are not separate processes, but stages of the same process so that interdisciplinarity is the "middle" in the triple sense of the term: the spread or margin, the spacing or interval, and the happy medium aimed at in the interpretation

The problem thus of improving the teaching and learning of mathematics in this way does not arise in terms of utility or use of mathematics to other fields of human activity, but in terms of the complementarity and originality of mathematics in the context of a meaningful activity, or an interdisciplinary project in a scholarly, democratic context of cooperation.

• What are the advantages and disadvantages entailed for mathematics in interdisciplinary approaches?

• What challenges does this raise for students and teachers?

• How can mathematics interact with other disciplines to support the understanding of a multi-dimensional problem, to see a complex phenomenon?

Sub-theme 2

Logic(s) when doing (performing) mathematics

Reasoning and proof is one of the core standards that appear in most mathematics curricula among various countries. We often ask our students to “show”, “justify”, “explain”, or “prove” an answer to a particular question “drawing on the first principles.” These actions lead to degrees of proof (Tall, Yevdokimov, Koichu, Whiteley, Kondratieva & Cheng, 2011). David Tall and his colleagues (2011) chose three definitions of proof (see table 1) to illustrate not only the importance of proofs in mathematics, but also the role and nature of proofs within the mathematical community.

Arnold, 2000 (p. 403)	Proofs are to mathematics what spelling (or even calligraphy) is to poetry. Mathematical works do consist of proofs, just as poems do consist of characters.
Rav, 1999 (p. 5)	‘Ordinary mathematical proofs’ –to be distinguished from formal derivations –are the locus of mathematical knowledge. Their epistemic content goes way beyond what is summarised in the form of theorems.
Bass, 2009	The truth of a mathematical claim rests on the existence of a proof. Stated this way, such a criterion is absolute, abstract, and independent of human awareness. This criterion is conceptually important, but practically useless.

Table 1. Three concepts of proofs. Source: Tall, Yevdokimov, Koichu, Whiteley, Kondratieva & Cheng (2011)

Drawing on these notions of proof, the authors clearly distinguish between formal proof (in the sense of Hilbert, for example) and ordinary mathematical proof, which leaves some room for the use of what they call “conceptual bridges” rather than
“explicit logical justification” between parts of a mathematical argument (Rav, 1999). From this [epistemological] point of view, proof involves “thinking about new situations, focusing on significant aspects, using previous knowledge to put new ideas together in new ways, consider relationships, make conjectures, formulate definitions as necessary and to build a valid argument.” (Tall et al., 2011,, p. 14)
The difficulty is that when someone starts to learn mathematics, s/he may be not used to think deductively from the very first moment. Children are used to face real-world situations, based on visual, hearing, touch perception, and acting on objects in the world. The problem here is that mathematics involves its own language, which may not correspond to the real-world one. Guzmán (2003) claimed that sometimes mathematics may look similar to the language that we use in our everyday life. But, according to him, this is false: mathematics has a peculiar language. The language of mathematics is characterized by being precise and unambiguous.
The discussion about what does it mean to “know mathematics” has given rise to various theories from an epistemological point of view. Prominent philosophers (such as Aristotle, Descartes, Locke, Berkeley, Hume, Kant, Poincare, Pascal, Leibniz, Frege, Russell, Hilbert, and others) have argued whether mathematics can be reduced to a set of primitive axioms from which mathematics is constructed by deduction (using syllogisms); or if mathematics emerge from intuitive perception that we then check using the language of the syllogism. Is all mathematics reducible to a set of primitive axioms, as Russell pretended? Is it the case that there are two types of claims: analytical claims (which can be deduced only from definitions using the principles of logics), and synthetic claims (which may involve data and procedures different from the definitions and the principles of logic), as Kant stated in *Critique of pure reason*? Such discussions not only have promoted a further development and understanding of mathematics, but also have had a great impact on how we teach mathematics, and the role proof plays in the learning of mathematics.
After more than half a Century of *Math Wars* (Schoenfeld, 2004), the fact is that mathematics education has been influenced by authors from different disciplines, forcing us to oscillate between

formalism and intuitionism. Piaget (1961), for example, thought that children pass through various stages of development, from the sensori-motor, through concrete operations, to formal operations. Harel and Sowder (1998) describe the cognitive growth of proof in terms of the child' development of *proof schemes*, described by Tall and his colleagues as relatively stable cognitive/affective configurations responsible for what constitutes ascertaining and persuading an individual of the truth of a statement at a particular stage of mathematical maturation. Van Hiele (1986) is another example of a didactical approach (in this case towards geometry) based on the development of Euclidean *Elements* using formalism (related to age maturity). According to them, children move from perceptual, intuitive reasoning, to a coherent framework of Euclidean deductive proof. Many others also draw on a deductive approach of mathematics to elucidate how children learn: using multiple representations operating in different ways (Golding, 1998, Duval, 2006); combining visual-spatial modes of operation with sequential symbolic ones (Paivio, 1991); using different modes of communication –enactive, iconic, symbolic– to develop sub-categories [structures] of number and logic (Bruner, 1966); etc.

In 1962 around seventy five teachers from different universities signed a letter published in *American Mathematical Monthly* and *The Mathematics Teacher* where they claimed:

> **Knowing is doing**. In mathematics, knowledge of any value is never possession of information, but "know-how." To know mathematics means to be able to do mathematics: to use mathematical language with some fluency, to do problems, to criticize arguments, to find proofs and, what may be the most important activity, to recognize a mathematical concept in, or to extract it from, a given concrete situation. (AA.VV., 1962; in Kline, 1973, p. 132).

However, what does it mean? Is mathematics learning a process of children developing their innate capacity of reasoning (using definition and deduction)? What is the role of intuition? Do we have any evidence to state that there is a connection between *perceptual recognition*, *verbal description*,

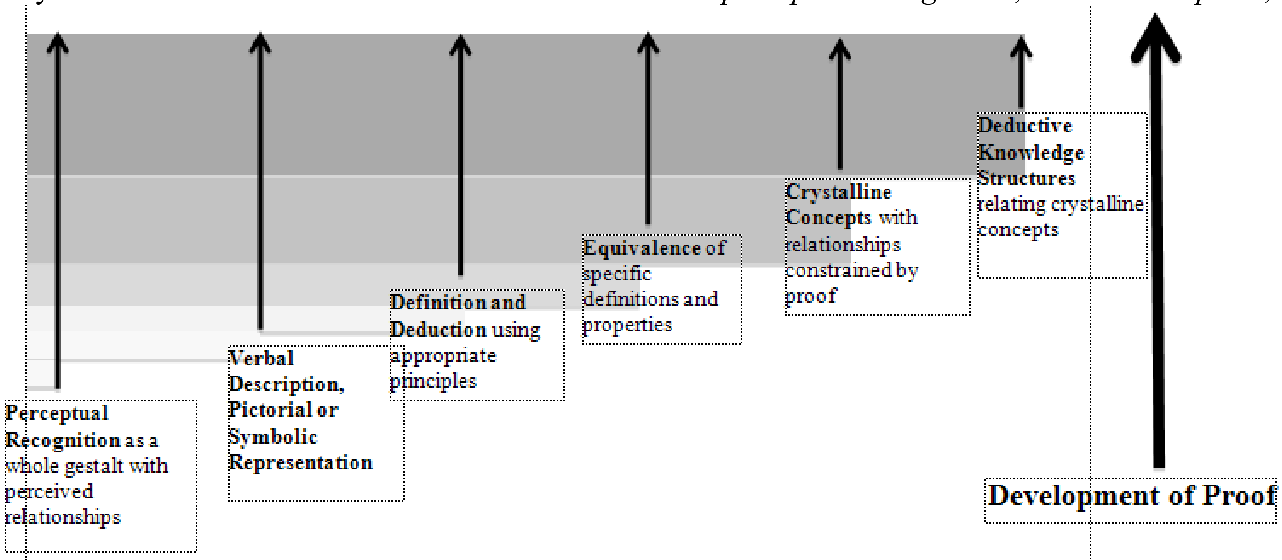

Figure 1. Development of proof (Tall et al. 2011).

and *definition and deduction* as suggested by Tall et al. (see figure 1)? Which kind of connection? Through the sub-theme of logic (or logics?) mathematical practices are questioned both in the relationships between reality and truth, and also in the role of logic in mathematics learning. Logical questions are part of mathematics in that they are based on the practice of reasoning involved in mathematical discovery (Heinzmann, 2013). In processes of research in mathematics or in the mathematics classroom, common logic is based on or confronted by mathematical logic. It is also necessary to consider the contributions of the logical analysis of language for research in mathematics education, which enriches *a priori* and *a posteriori* analyses (Durand-Guerrier, 2013). This sub-theme will also address both the practical and theoretical aspects of the teaching and learning of proof and demonstration, especially in the relationship that may exist between argumentation, logic and evidence (Hanna & al., 2012). Activities stimulating the development of logical thinking and the obstacles in the learning process related to logic will be discussed from both a scientific (epistemological) and a didactic point of view.

- What is role of logic in reasoning?
- What are the links with the arguments, evidence?
- What teaching could allow the acquisition of reusable logic skills?
- Is it necessary to include a course on logic in university teacher training (why, under what conditions, etc.)?

Sub-theme 3

Realities, technologies and mathematical experiences

Reflections by Artigue (2000) on the problems of using technology in the mathematics classroom report that after 30 years of use, we are far from solving the problem of teaching mathematics in a technology environment. At the time, she was already thinking about issues of:

1. The poor educational legitimacy of computer technologies as opposed to their social and scientific legitimacy;
2. The underestimation of issues linked to the computerization of mathematical knowledge;
3. The dominant opposition between the technical and conceptual dimensions of mathematical activity;
4. The underestimation of the complexity of the instrumentation processes. (pp. 8-9)

Today, after 30 years of use of technology in the mathematics classroom, the problem continues to be a topic of discussion (Aldon et al, 2008, 2010; Artigue, 2002; Guin & Trouche, 1999, 2002; Gravemeier, 2012; Hoyles, Noss,& Kent, 2004; Hitt & Kieran, 2009; Hitt, 2007, 2010; Karsenti & Collin, 2002; Kieran & Guzmán, 2010; Lagrange, 2000, 2003).

Technology is advancing at an incredible rate while its use in the mathematics class is at "a snail's pace." For example, while the Interactive Whiteboard and electronic computer tablets have made their appearance in schools and are beginning to be widely used, some researchers are reflecting on the relevance of these technological tools in schools. For example, Noël & Marissal (2012), citing the Karsenti Report:

> According to Karsenti, director of the Canada Research Chair on information technology and communication in education, there is no independent study to justify the massive and rapid purchase of interactive whiteboards. "The only studies of TBI (interactive whiteboards) have been subsidized by the manufacturers..." (The Press, March 1, 2012, Quebec)

The controversy is then built on the importance of deciding whether to use interactive whiteboards or tablets as a teaching tool in the mathematics classroom in Quebéc, even when these things are an integral part of the everyday lives of students at all levels.

The renewal of science education has been the subject of discussion in recent years. There are projects (e.g., PRIMAS, Fibonacci, EdUmatics) for the production of interdisciplinary activities devised and carried out by educationalists (in mathematics, physics, chemistry, and biology). This approach involves training students in the processes of mathematical modeling (Gravemeier, 2007) and the resolution of problems and / or problem situations. Explicitly, this approach leads to a reflection on the role of technology in solving problems (Aldon, 2009) and the role it can play in the process of mathematical modeling (Hitt, 2013). It seems that in the process of mathematical modeling, the paper-and-pencil environment is important in the early stages to organize actions that students will later carry out with the use of technology.

- Different realities, sensitivities, experiments, objectives, in themselves, could be defined and built. What reality corresponds to mathematical productions and inventions?

As part of this, if we think about the type of performances that technology allows, we can ensure that they are generally classified as institutional representations (representations that can be found in curriculum, textbooks, etc.). On the other hand, the representations that students express in a paper-and-pencil approach can be completely different institutional representations (diSessa et al, 1991; Hitt, 2013). From this perspective, it seems natural, then, to be faced with different ways of using mathematics in relation to the work environment. This leads us to reflect on the importance of creating activities that can promote reconciliation between the pencil-and-paper production and the technological production (Hitt & Kieran, 2009; Kieran & Guzmán, 2010). We could ensure that the continuing evolution of electronic artefacts, with the integration of electronic whiteboards in the classroom, and actions from a paper-pencil environment, are incorporated into actions with the computer/tablet, allowing the articulation between the institutional representations (Duval, 1993, 1995) through an evolution of functional representations in a collaborative learning activity in the mathematics class (Hitt, 2013).

- Is the mathematics used by a student to solve a mathematical task in a paper-and-pencil environment different from the one he or she would use in a technological context?

The term "investigative approach" is rather well used in the teaching of experimental sciences, even though research shows that the experimental part of mathematics is a lever for learning concepts (Gardes, 2013). Experimentations to renew science education in high school are part of an international context of disaffection for scientific courses (OECD, 2006) and aim to propose a modification to science education by engaging students in thinking about critical scientific problems, as evidenced, for example, in the special issue of ZDM: "Implementation of Inquiry -Based Learning in Day- to-Day Teaching" (Nov. 2013).

- But what about mathematics? Is it that the process of investigation and search for problems coincide? What is the place of experimental mathematics? What is the role of ICT in these experimental aspects?
- Experience in mathematics could perhaps be developed to naturalize or to find a concept. For a synthetic a priori concept to be worked on, it is necessary that the objects that lead to understanding be naturalized?

Sub-theme 4

Multiculturalism and reality

Since prehistoric times mathematical creations and advances in mathematics have been related to the realities faced by mankind from the point of view of number as geometric. Currently in several ethnic groups the consideration of number is different, depending on what you count (Bishop 1988, Smith 1994, Palascio 1995, Ezeife 1999). How to understand this prerequisite with the notion of equipotent sets ? The basis of counting may be different, and also in the mathematics class when several different cultures meet we can not ignore these differences and how to integrate them to allow students to acquire the basics of mathematics.

According to the same understanding, the acquisition of some basics may be hampered by the cultural reality of students. For example, for the Inuit, as explained by Louise Poirier (2005, 2006), the notion of an equal share does not make sense in the community because everything belongs to everyone, and people use things according to their needs, respecting the ethics. There is no question that someone will take something, especially if they do not need it.

- How under these conditions is it possible to understand the concept of Euclidean division?
- In the same way, the use of fingers to count varies greatly from one culture to another.

The same applies to representations of space and perspective in particular, there are many differences from one culture to another.

Many examples could be cited.

- How to use these differences to advance the class group in a multicultural context and enrich mathematical learning?

Teaching in multicultural classes realities can be addressed from the point of view of different social and ethnic cultures of students from a professional point of view. Indeed the resolution of some practical technical problems may require the establishment of different methods to link them with theoretical knowledge.

- How can teachers use these differences within the classroom to enrich the mathematical learning of students ?

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